



# Algorithms - Preliminaries

**Rubric: No Raw Loops**

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## Definition

“An *Algorithm* is a process or set of rules to be followed in calculations or other problem-solving operations, especially by a computer.” – *New Oxford American Dictionary*

# A Simple Algorithm

```
int r = a < b ? a : b;
```

- What does this line of code do?

# A Simple Algorithm

```
// r is the minimum of `a` and `b`  
int r = a < b ? a : b;
```

# A Simple Algorithm

```
int r = min(a, b);
```

# Minimum

```
/// returns the minimum of `a` and `b`  
int min(int a, int b) {  
    return a < b ? a : b;  
}
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  - Or implied by the preconditions of the algorithm
- The postconditions for the algorithm must follow from the sequence of statements

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Functions allow us to build a vocabulary focused on semantics.

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- adjectives: `empty` (ambiguous but used by convention)
- copular constructions: `is_blue`
- consider a verb if the complexity is greater than expected

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- `name()` not `get_name()`

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- `T&&`, where `T` is not deduced
- `T`, For known or expected small types and to avoid forward references

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*sink*: input range

*result*: output iterator

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- All other arguments are *let* (read-only, copied if escaped)
- Results of functions with names starting with "alloc," "new," "copy," or "create" are owned solely by the caller; other results are read-only

# Argument Types

```
void display(const vector<unique_ptr<widget>>& a) {  
    //...  
  
    a[0]->set_name("displayed"); // DO NOT  
  
    //...  
}
```

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## Meaningless objects

- A meaningless object should not be passed as an argument (i.e., an invalid pointer).

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erase(a, a[0]);  
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auto f = begin(a);  
a.push_back(5);  
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- Example: the reference returned from `vector::back()`

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- Examples: `swap()`, `exchange()`, `min()`, `max()`, `clamp()`, `toLowerCase()`...

A *non-trivial* algorithm requires iteration

- iteration may be implemented as a loop or recursion

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- A finite decreasing property where termination happens when the property is zero

The postcondition of the iteration is the above invariant when the decreasing property reaches zero

# Erase

```
template <class T>
void erase(vector<T>& c, const T& value) {
    c.erase(remove(begin(c), end(c), value), c.end());
}
```

# Remove

```
/**  
    Removes values equal to `a` in the range `[f, l)`.  
  
    \return the position, `b`, such that `[f, b)` contains all the  
            values in `[f, l)` not equal to `a` in the original order  
            values in `[b, l)` are unspecified  
*/
```

```
template <forward_iterator I, class T>  
auto remove(I f, I l, const T& a) -> I;
```

# Remove

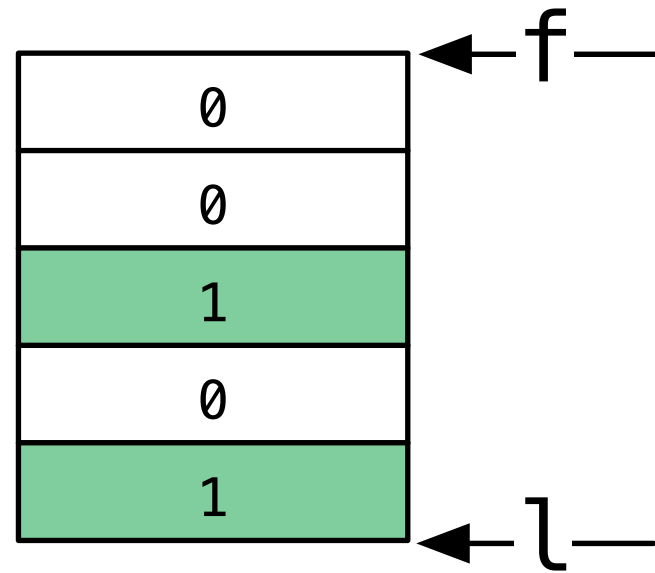
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a: 

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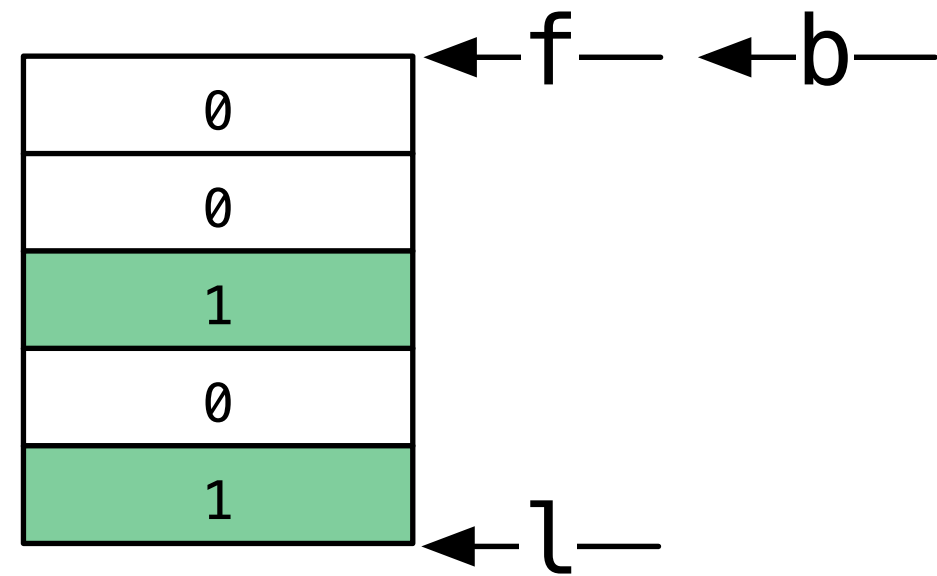


## Remove

```
template <forward_iterator I, class T>  
auto remove(I f, I l, const T& a) -> I {  
    auto b{find(f, l, a)};  
};
```

a: 

0
---



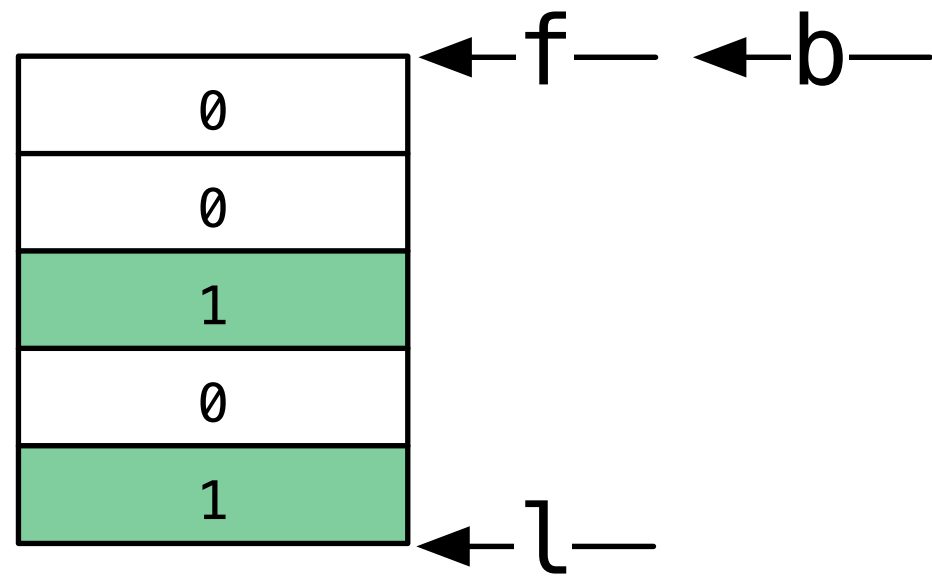


## Remove

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template <forward_iterator I, class T>
auto remove(I f, I l, const T& a) -> I {
    auto b{find(f, l, a)};
    if (b == l) return b;
}
```

a: 

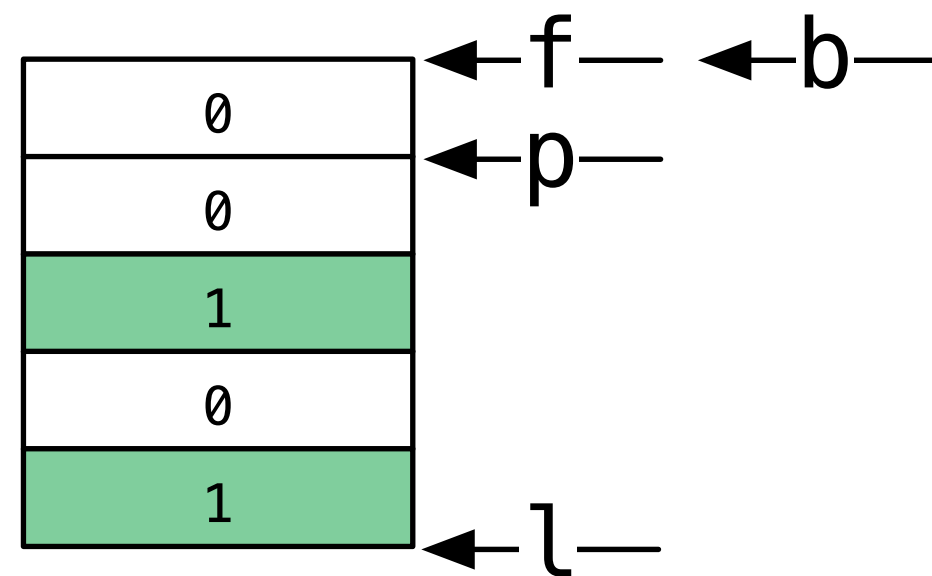
0
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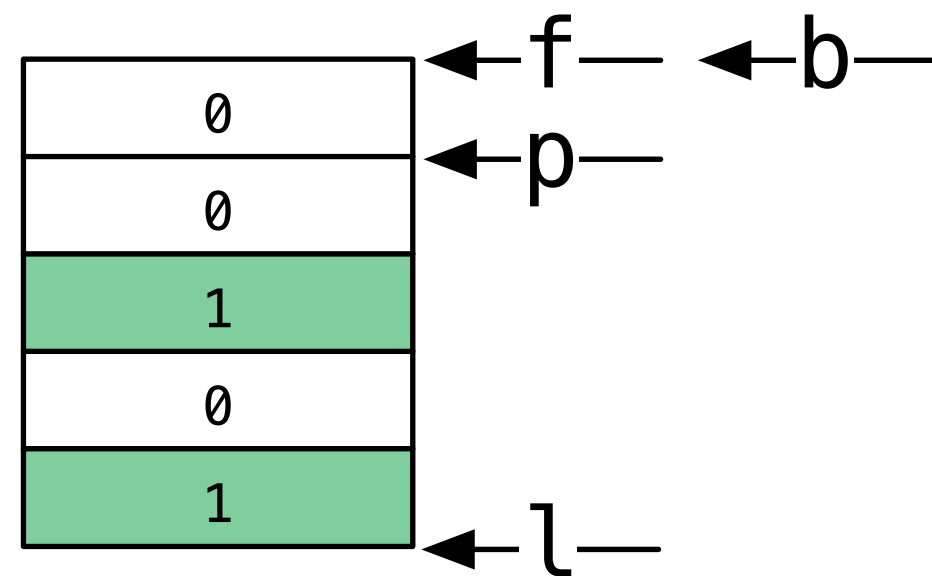


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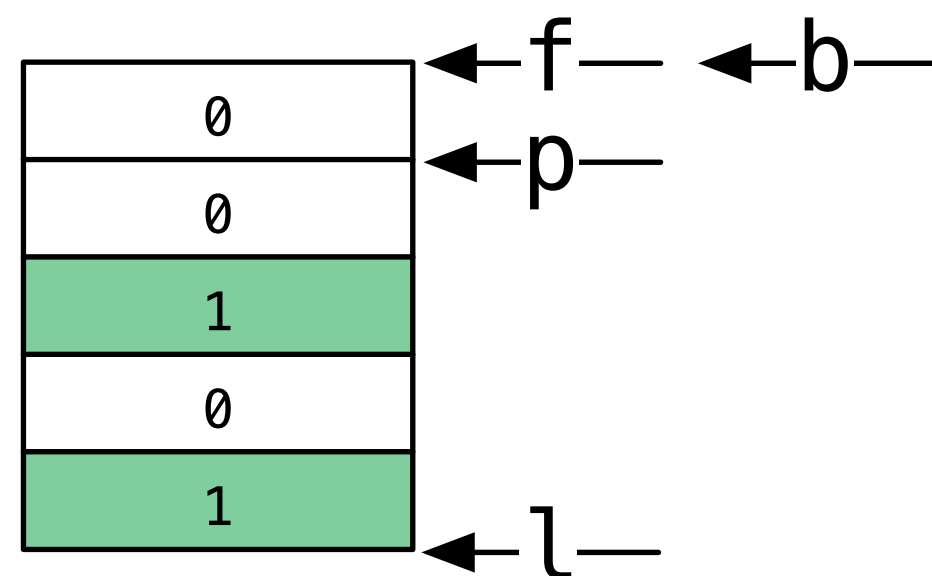


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    auto b{find(f, l, a)};
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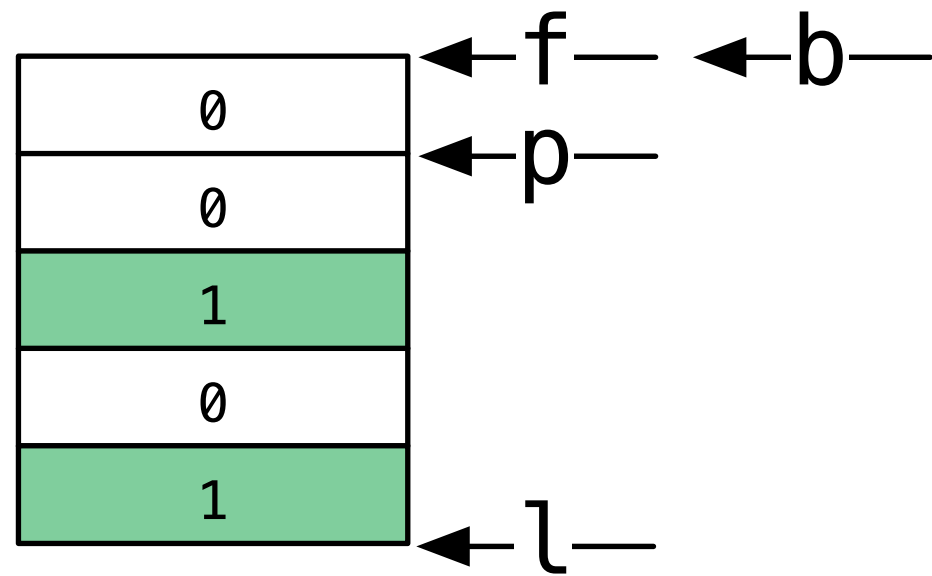


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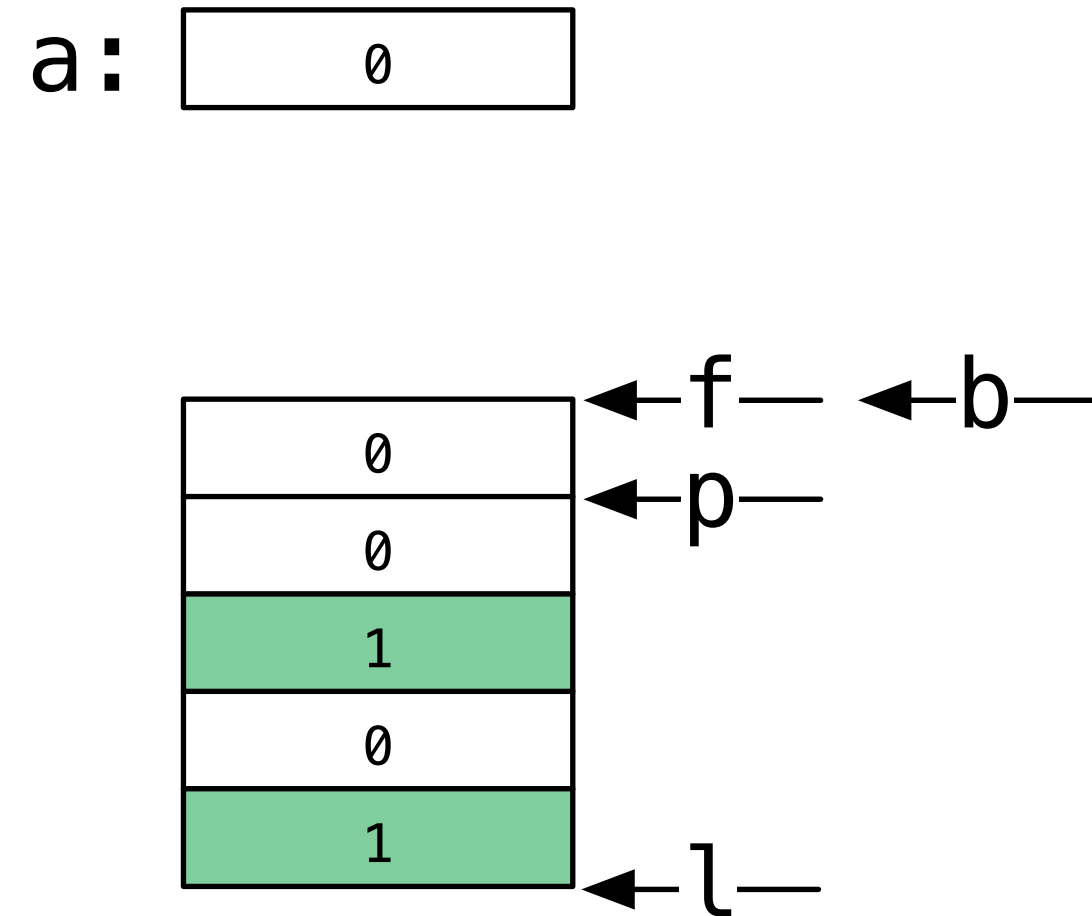
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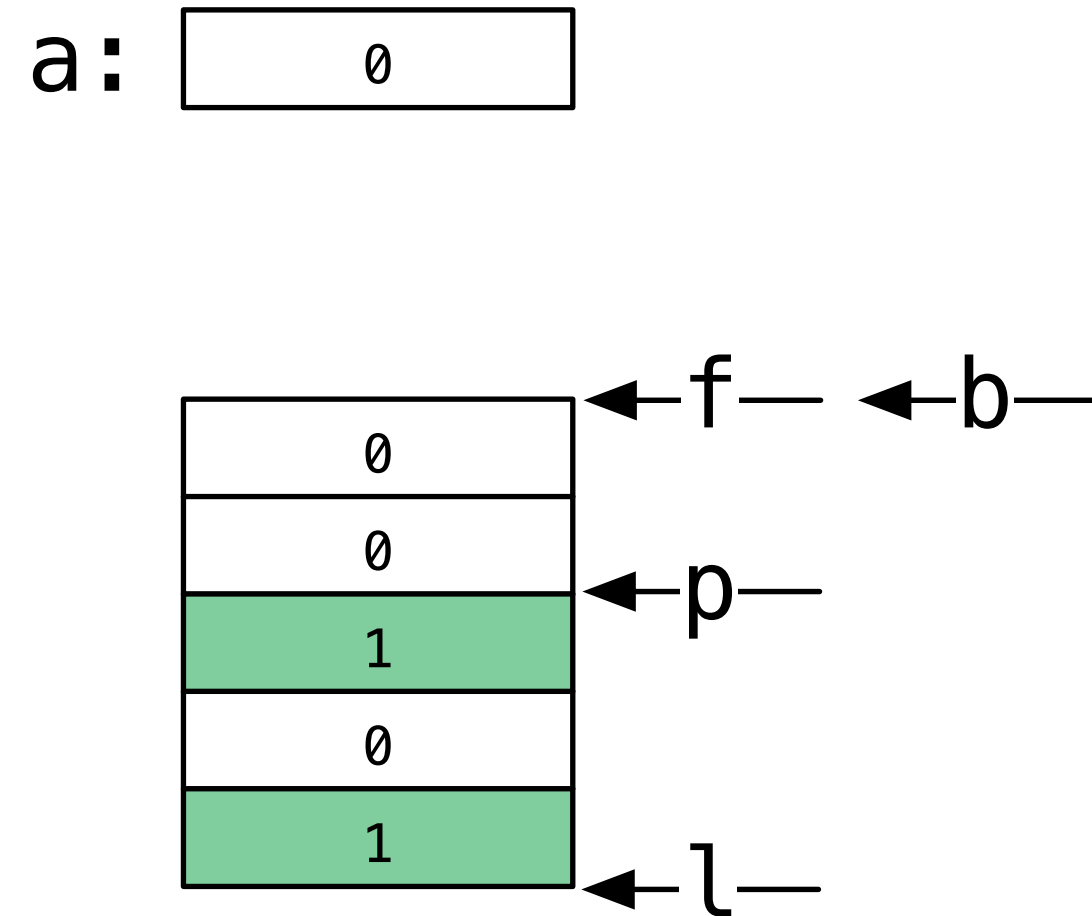
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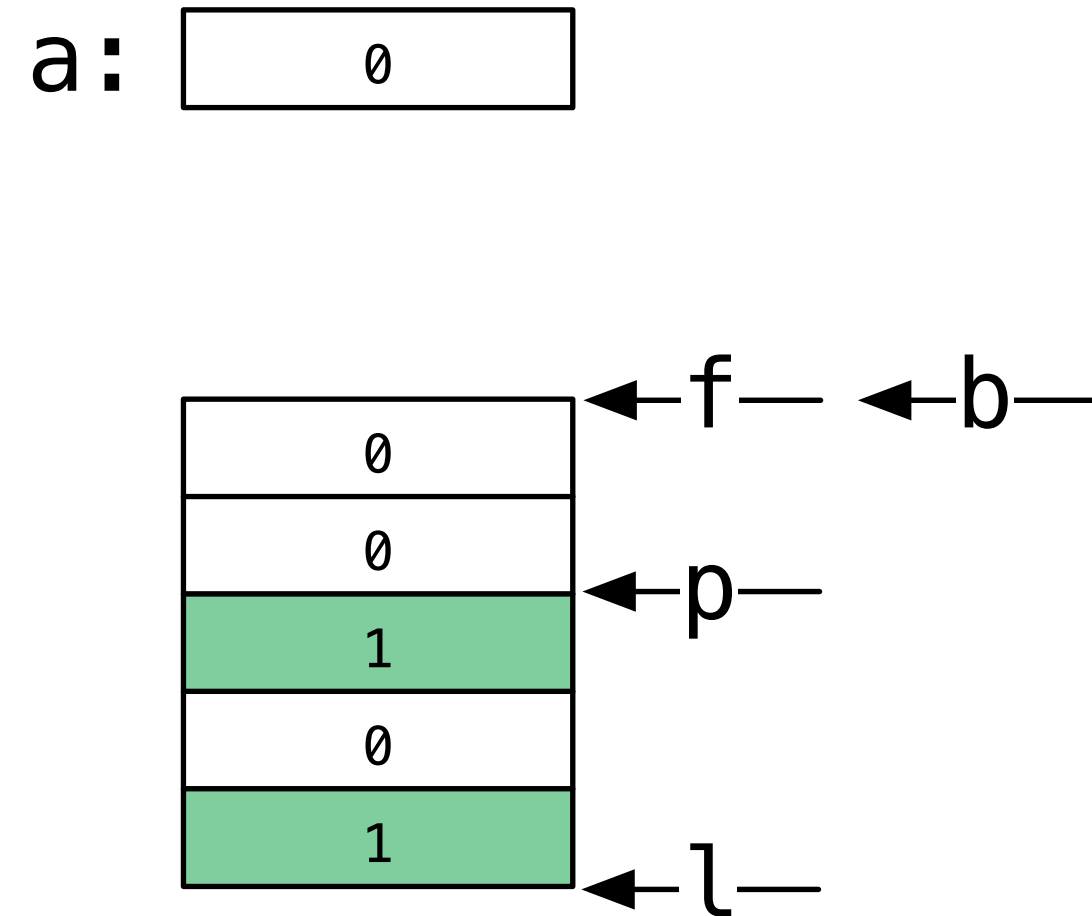
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    while (p != l) {
        if (*p != a) {
            *b = std::move(*p);
            ++b;
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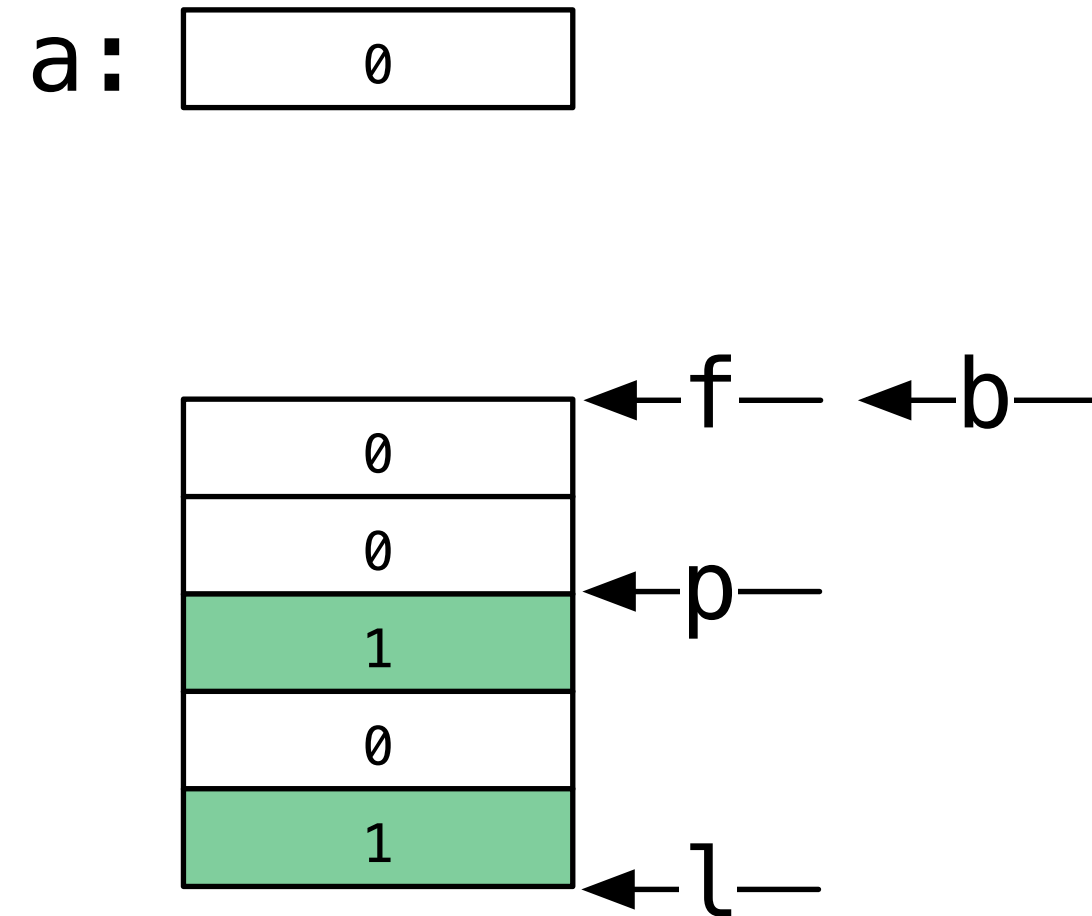
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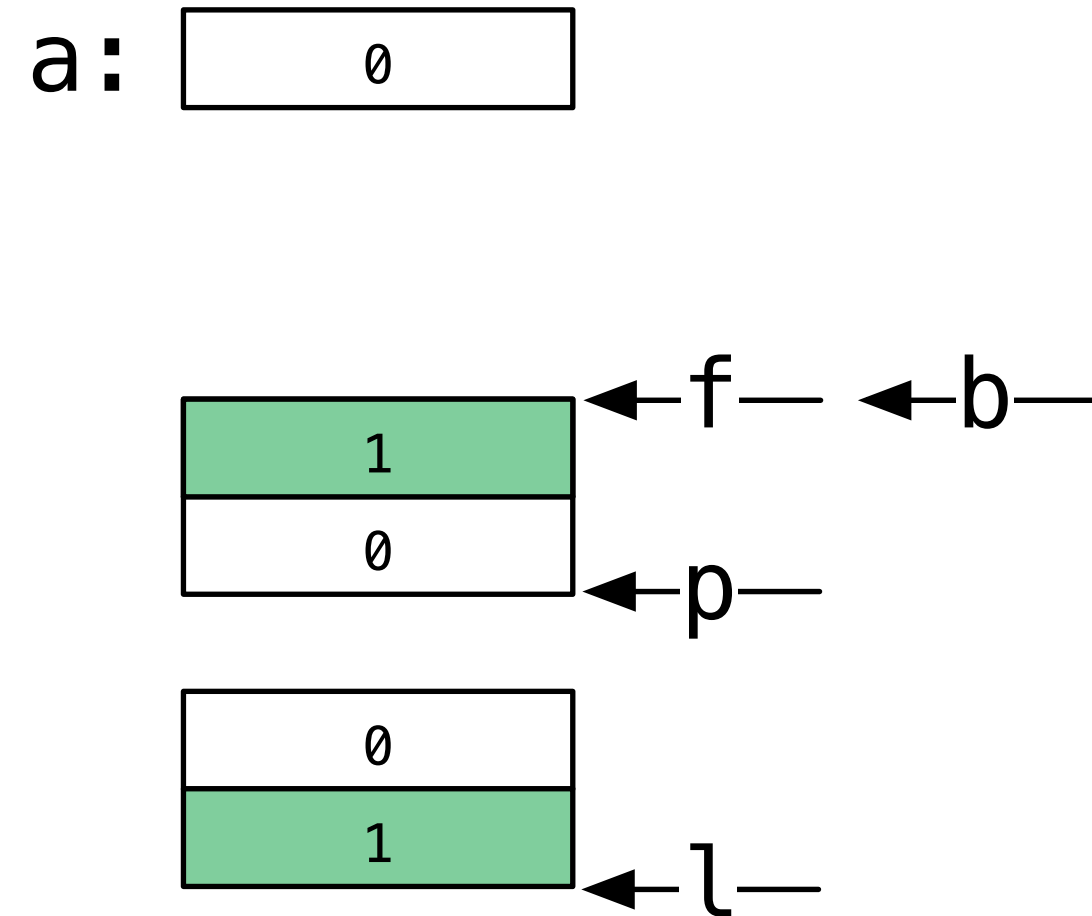


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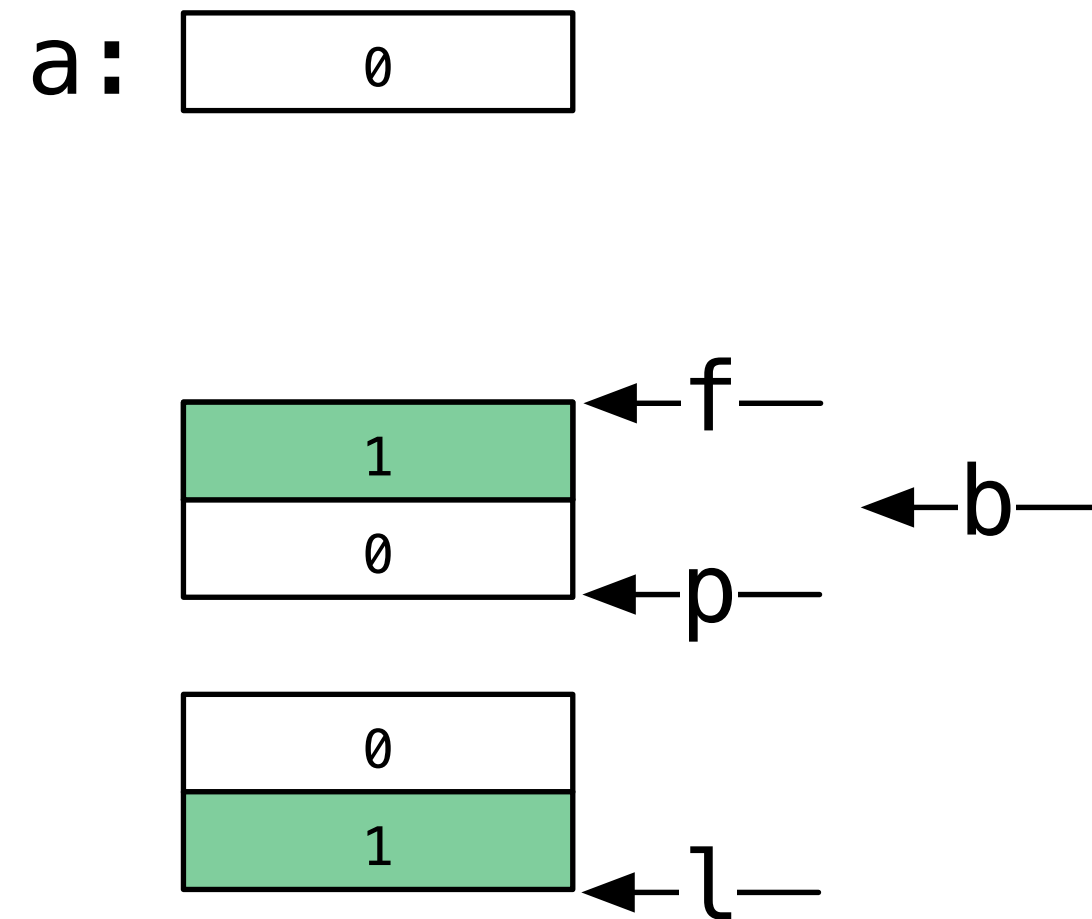
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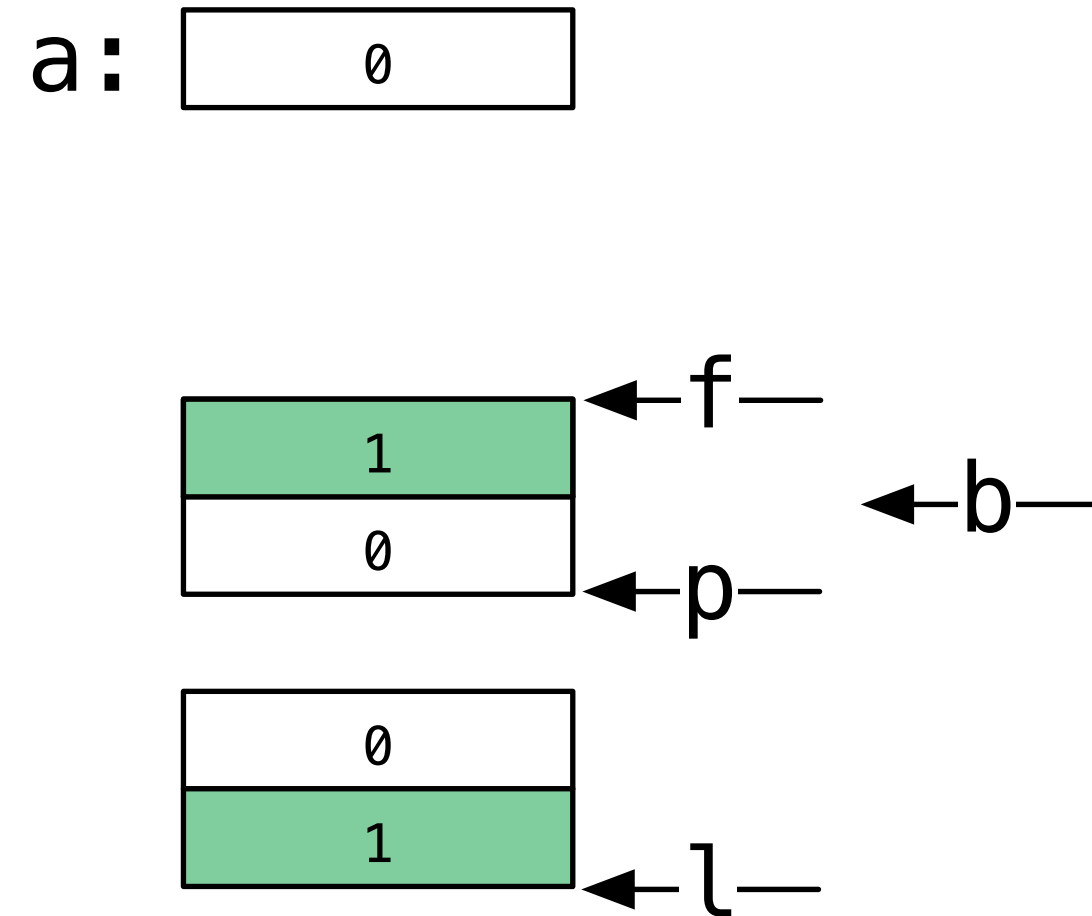
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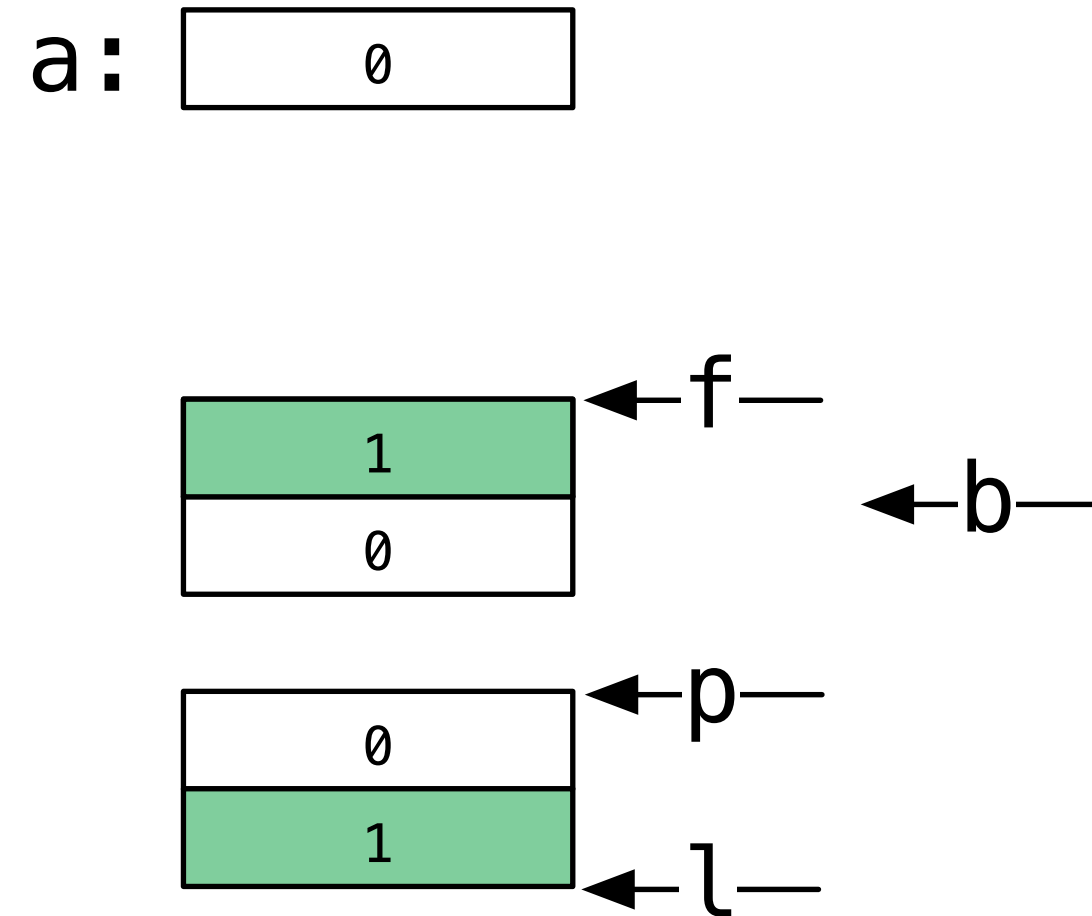
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    while (p != l) {
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            *b = std::move(*p);
            ++b;
        }
        ++p;
    }
}
```

## Remove



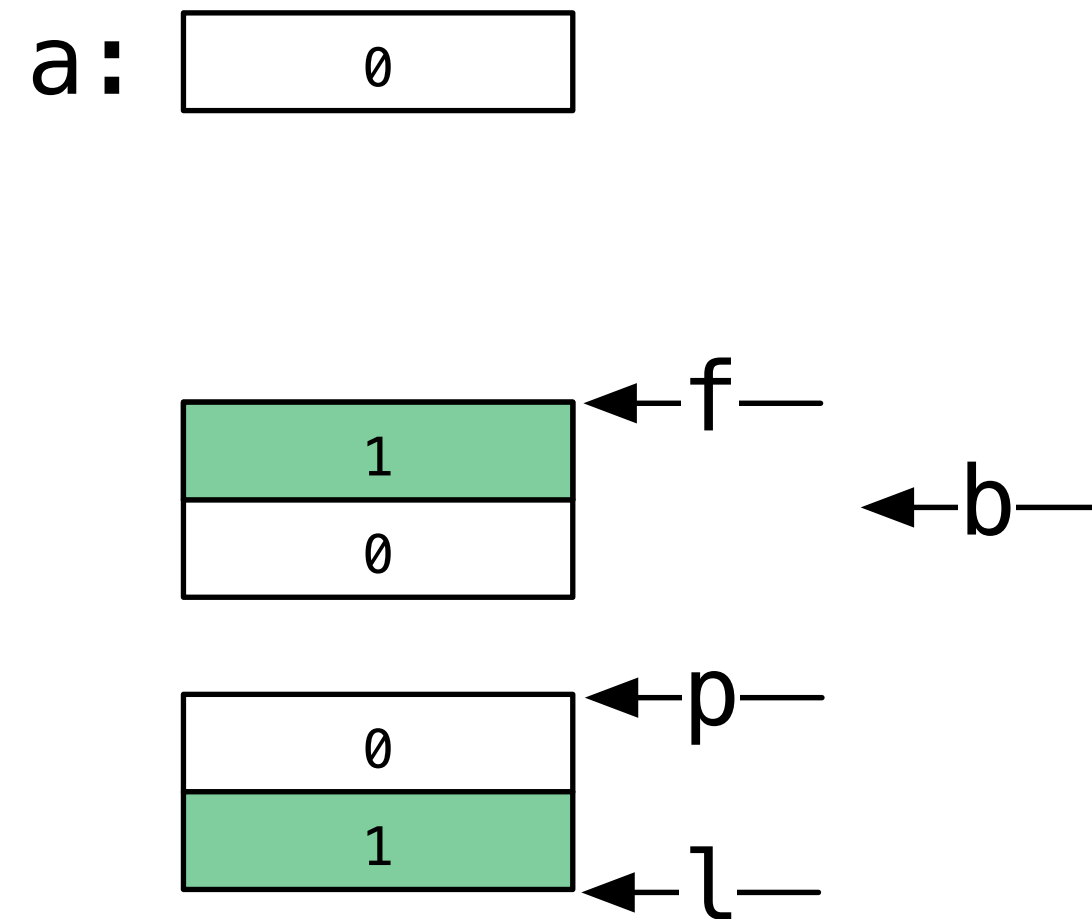
```
template <forward_iterator I, class T>
auto remove(I f, I l, const T& a) -> I {
    auto b{find(f, l, a)};
    if (b == l) return b;
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}
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# Remove



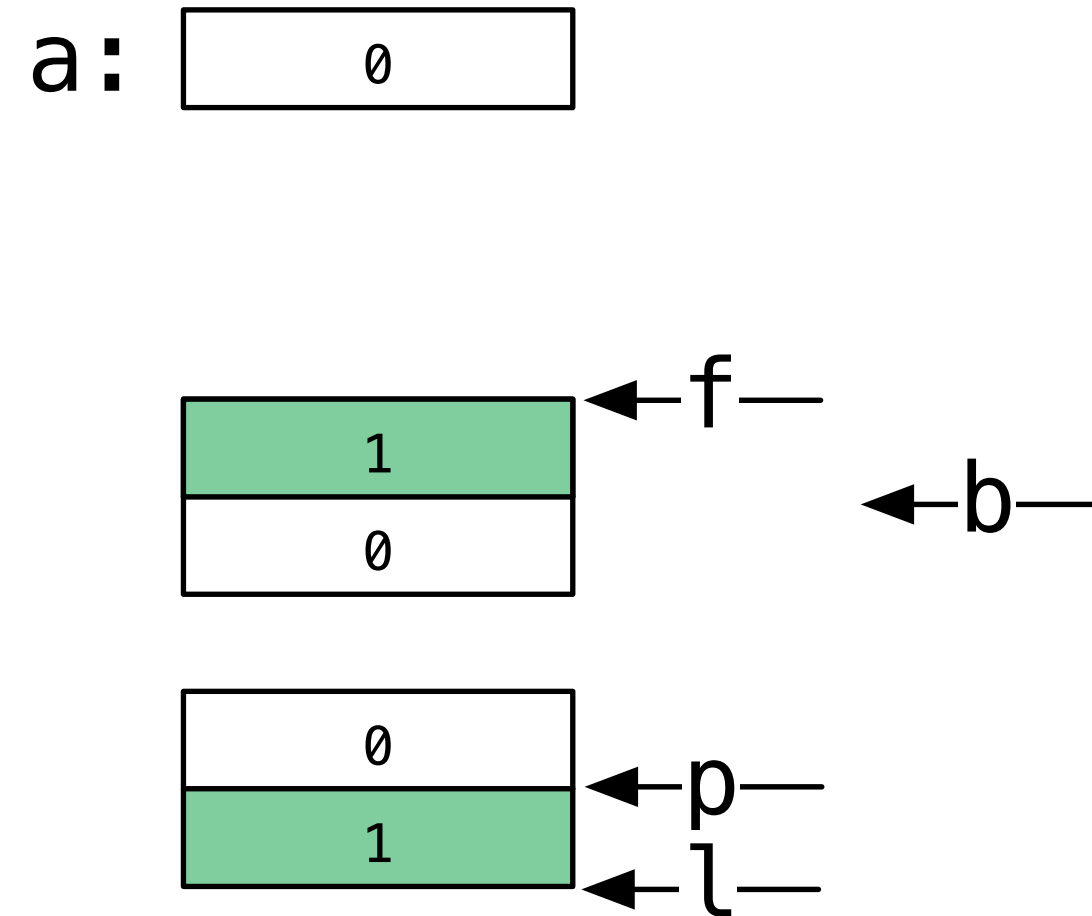
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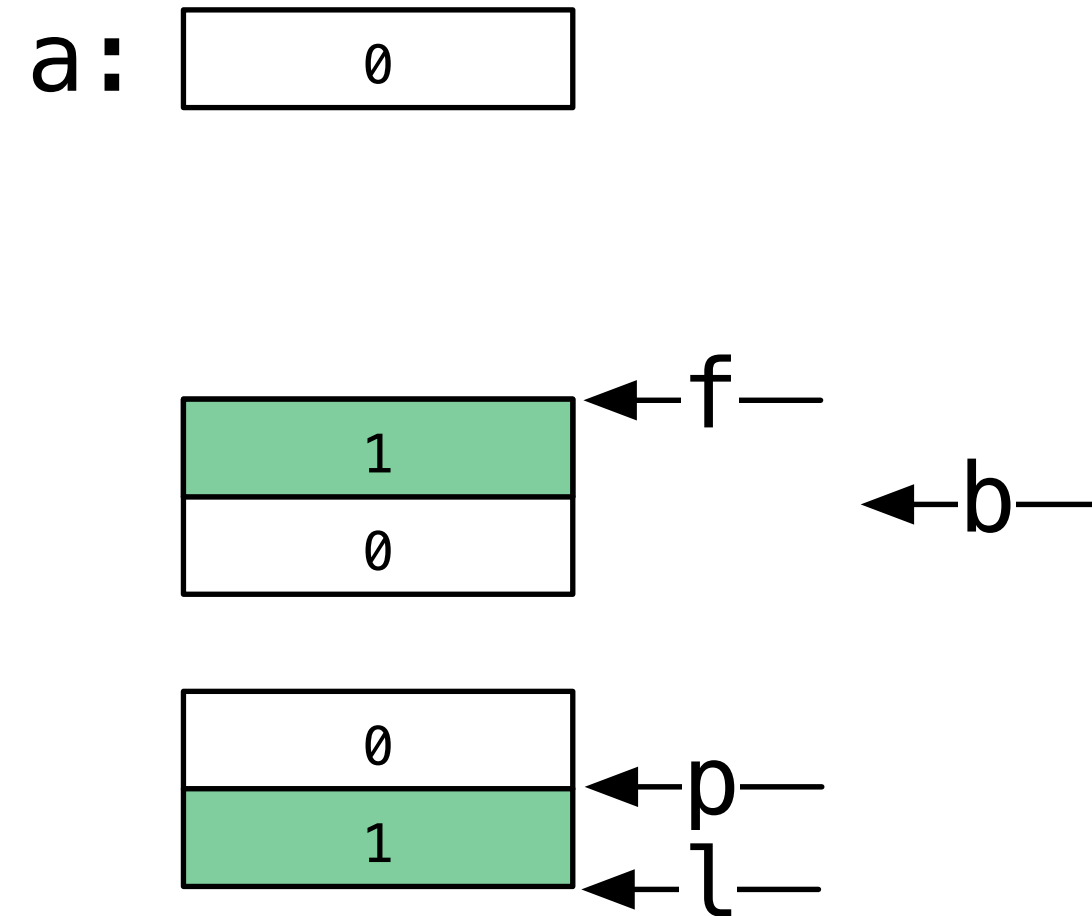
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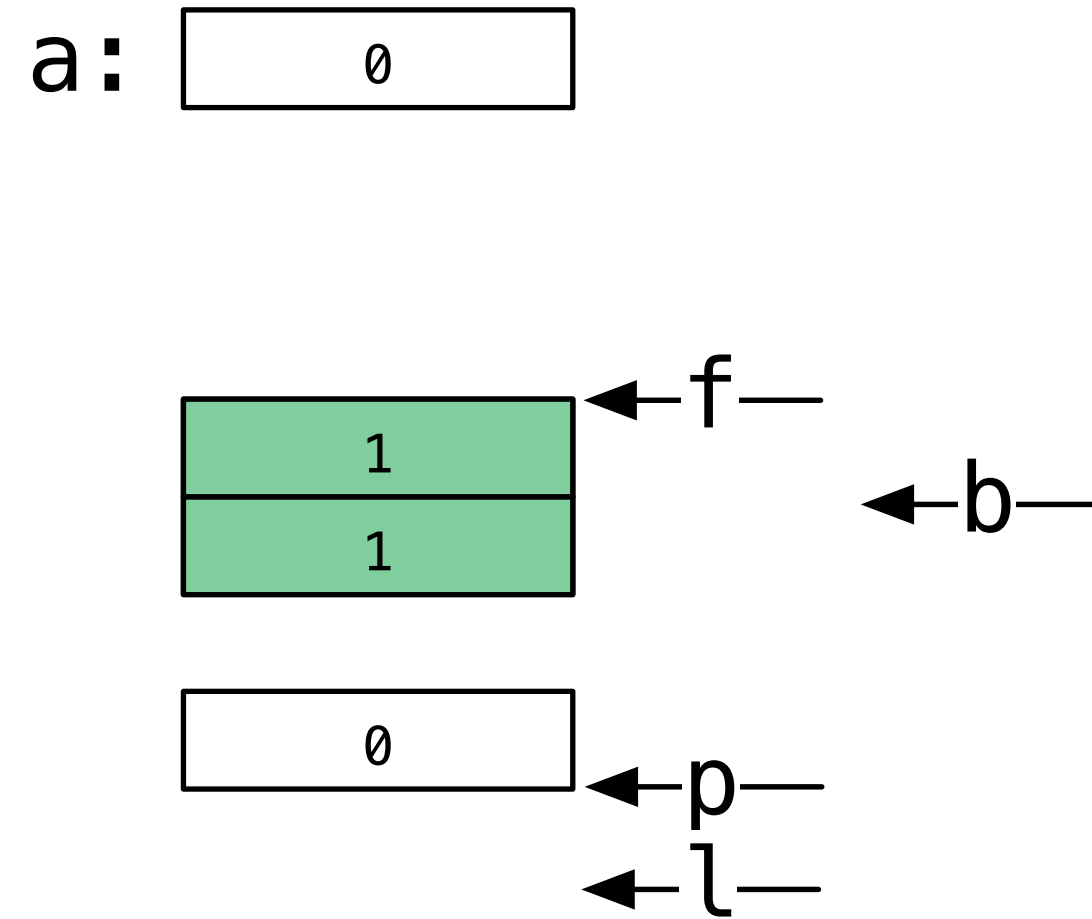
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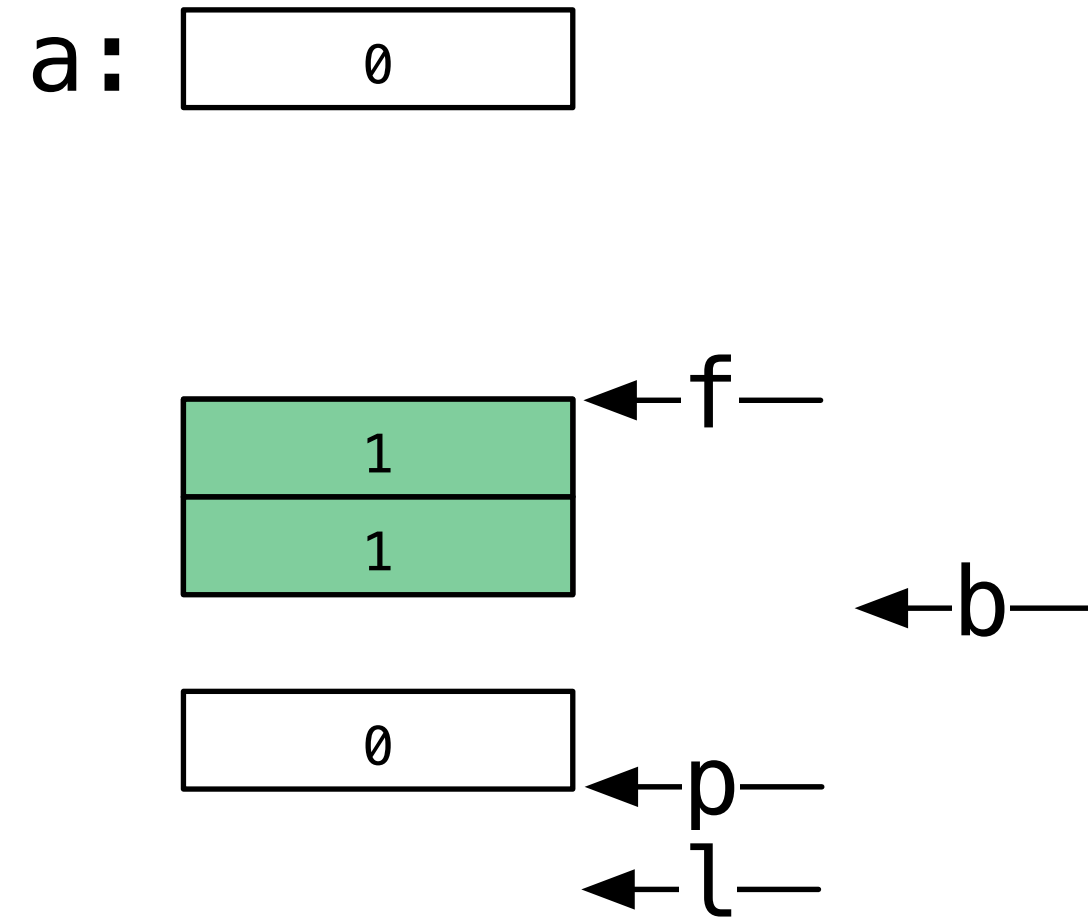


## Remove



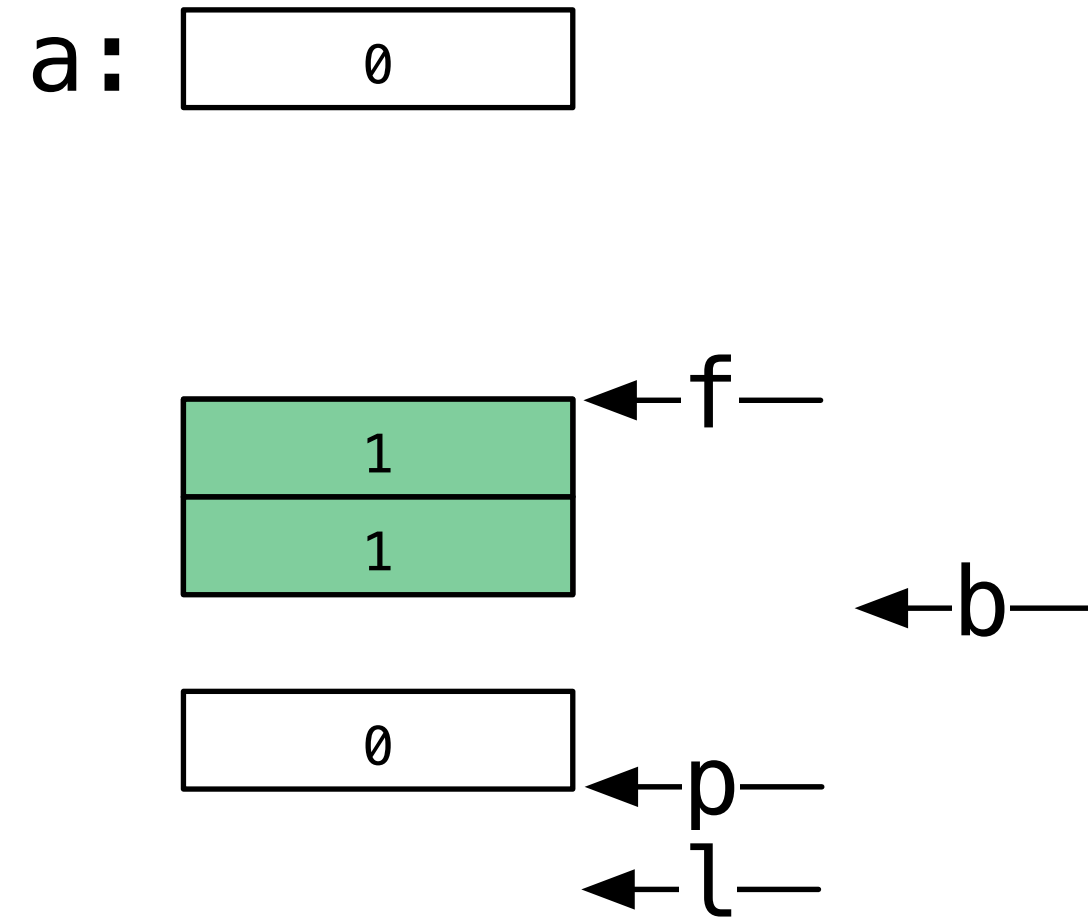
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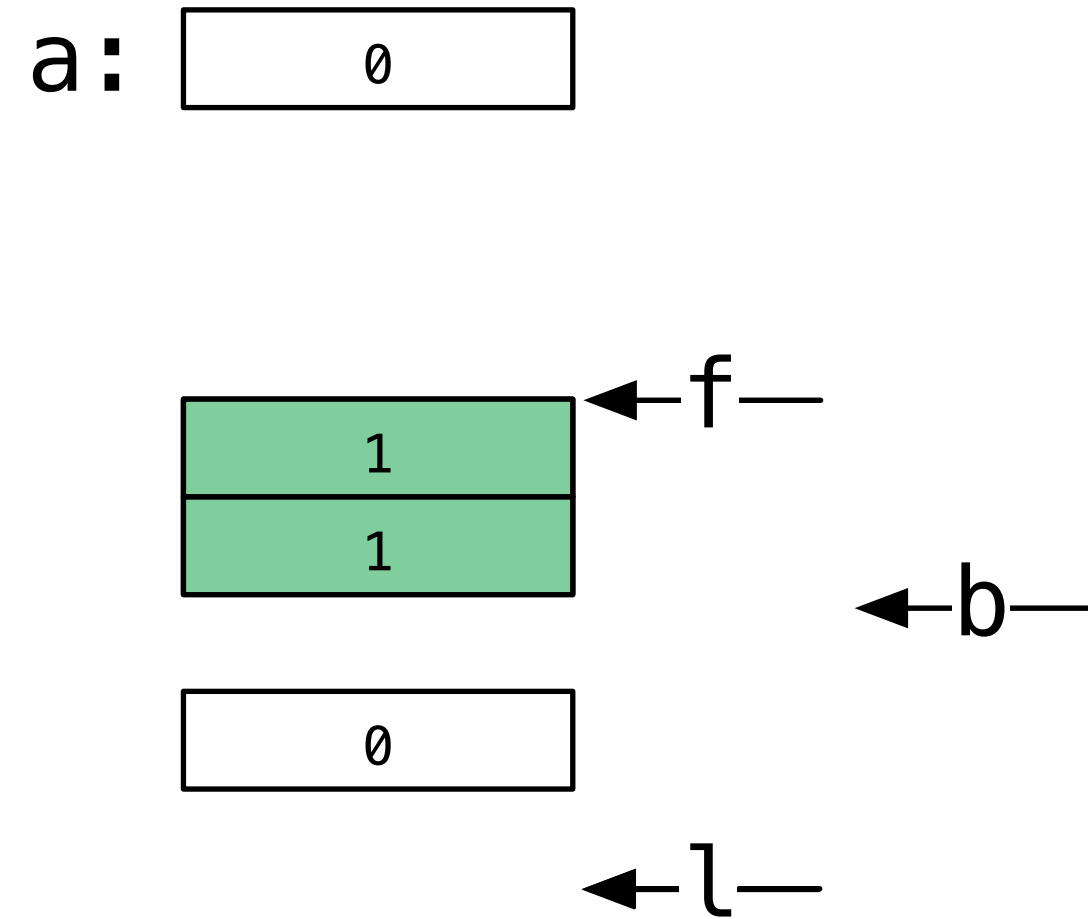
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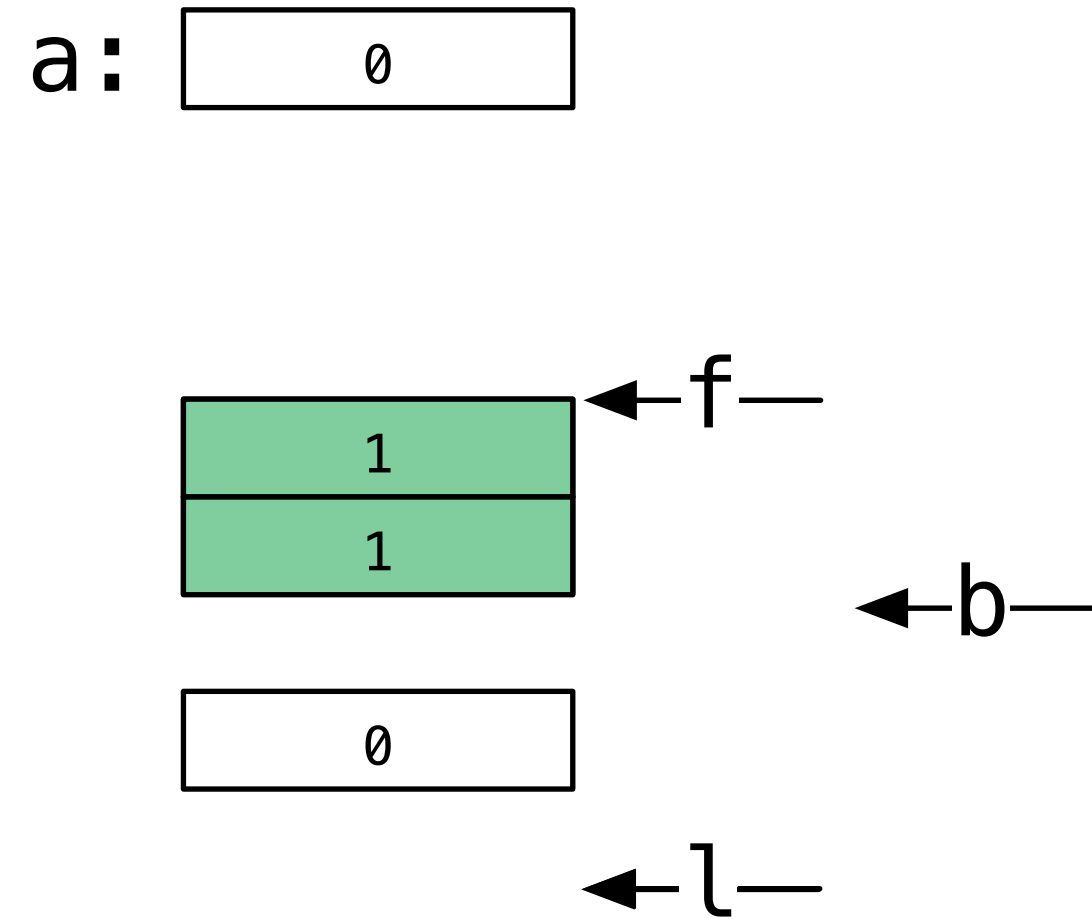
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}
```

## Remove



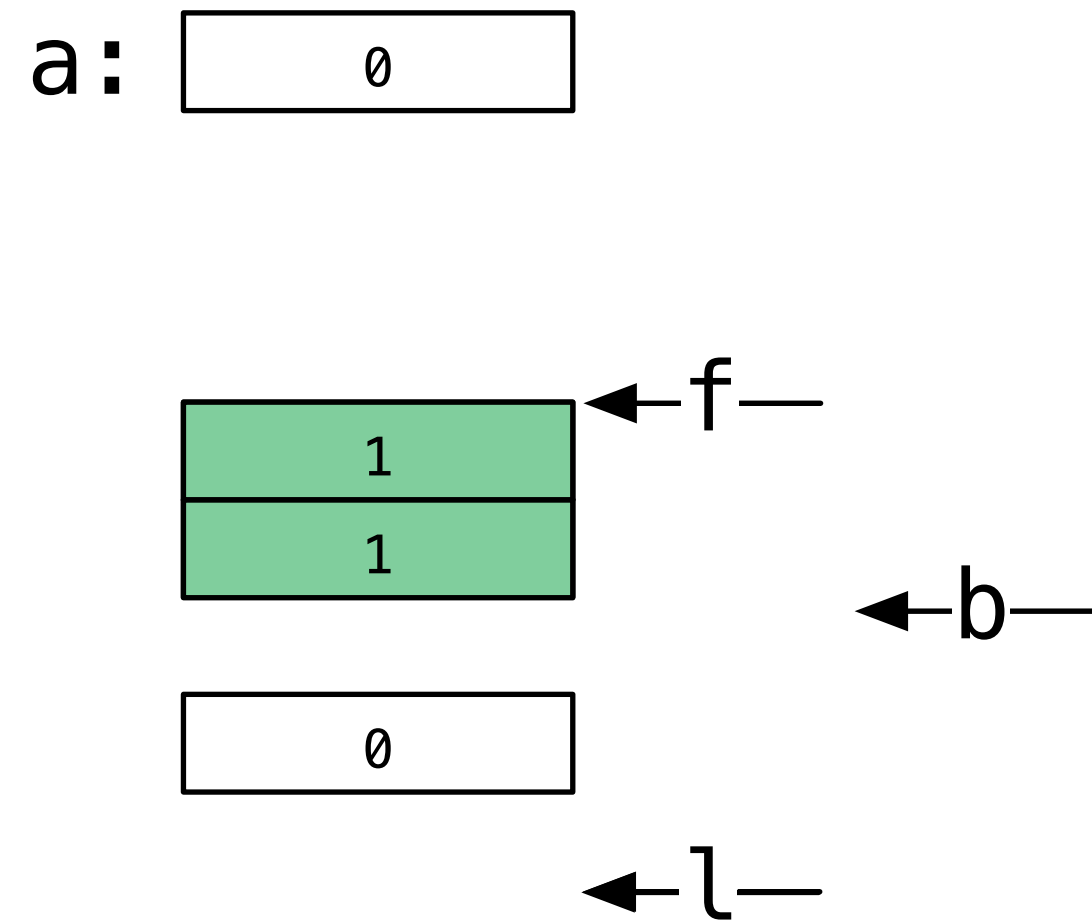
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    Removes values equal to `a` in the range `[f, l)`.  
  
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*/
```

```
template <forward_iterator I, class T>  
auto remove(I f, I l, const T& a) -> I;
```

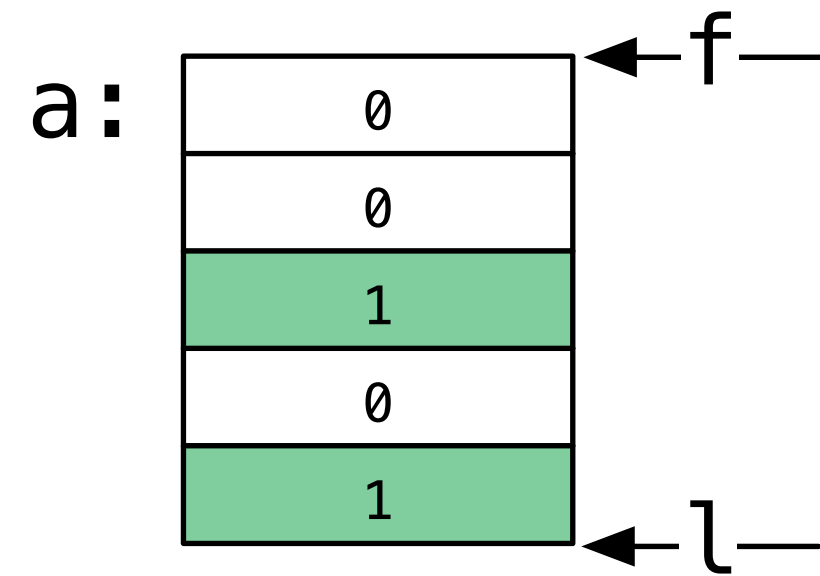
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*/  
  
template <forward_iterator I, class T>  
auto remove(I f, I l, const T& a) -> I;  
  
vector a{0, 0, 1, 0, 1 };  
erase(a, a[0]);
```



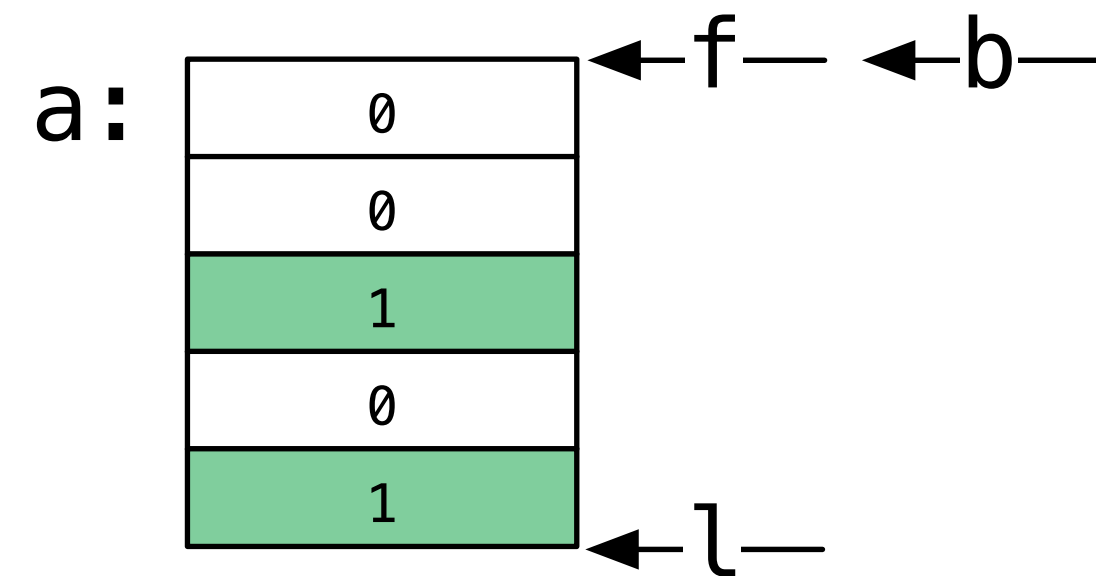
## Remove

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template <forward_iterator I, class T>  
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```



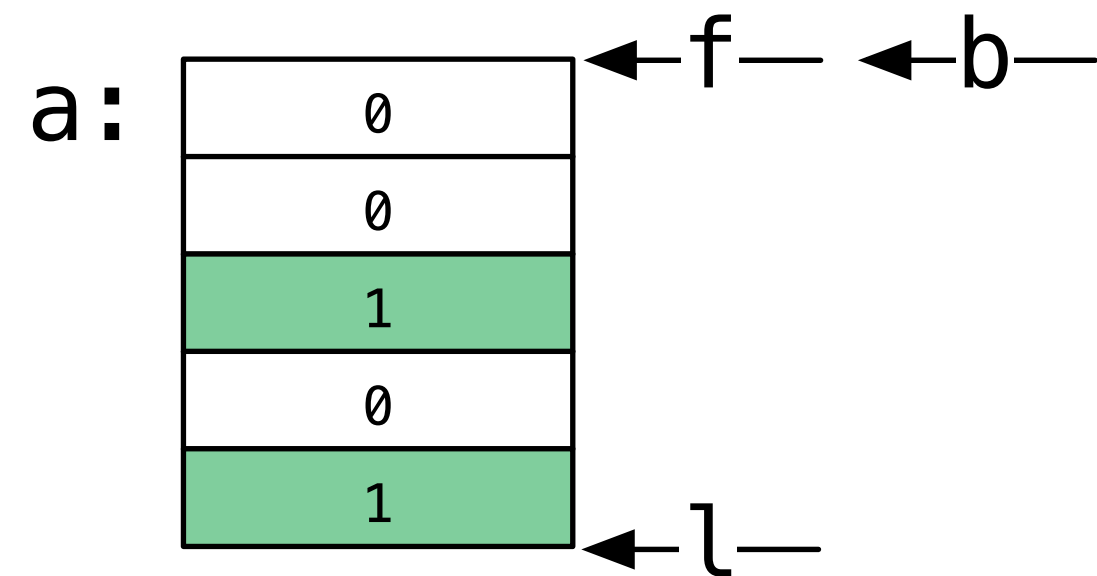
## Remove

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template <forward_iterator I, class T>  
auto remove(I f, I l, const T& a) -> I {  
    auto b{find(f, l, a)};  
    ...  
}
```



## Remove

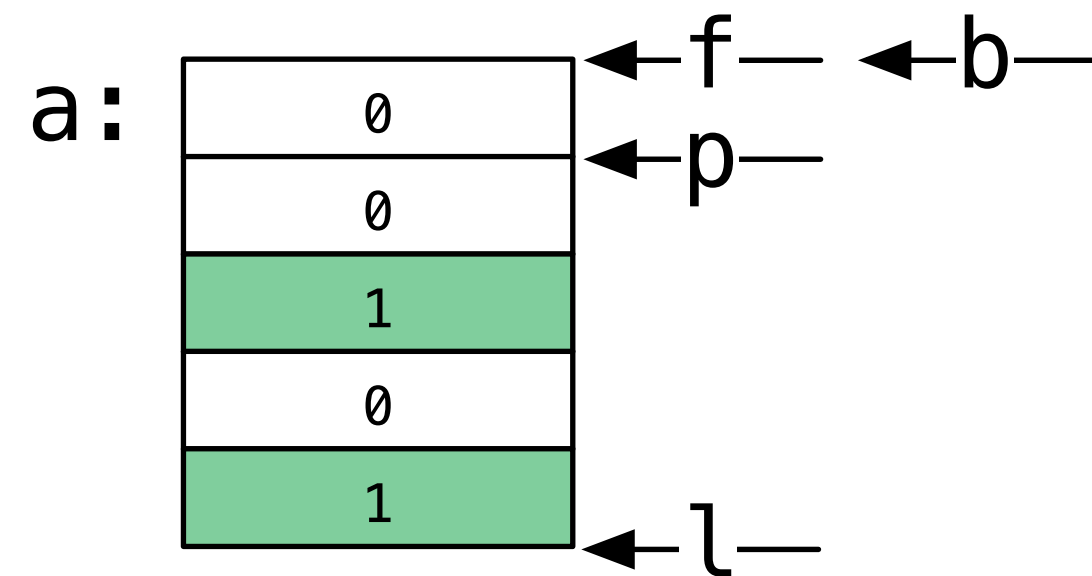
```
template <forward_iterator I, class T>
auto remove(I f, I l, const T& a) -> I {
    auto b{find(f, l, a)};
    if (b == l) return b;
}
```



## Remove

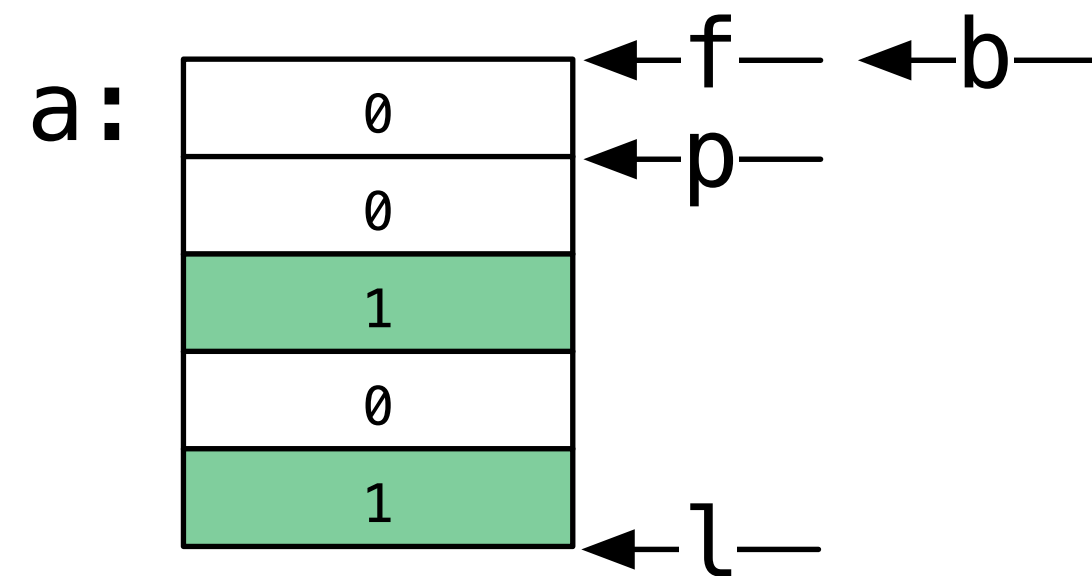
```
template <forward_iterator I, class T>
auto remove(I f, I l, const T& a) -> I {
    auto b{find(f, l, a)};
    if (b == l) return b;
    auto p{next(b)};

```

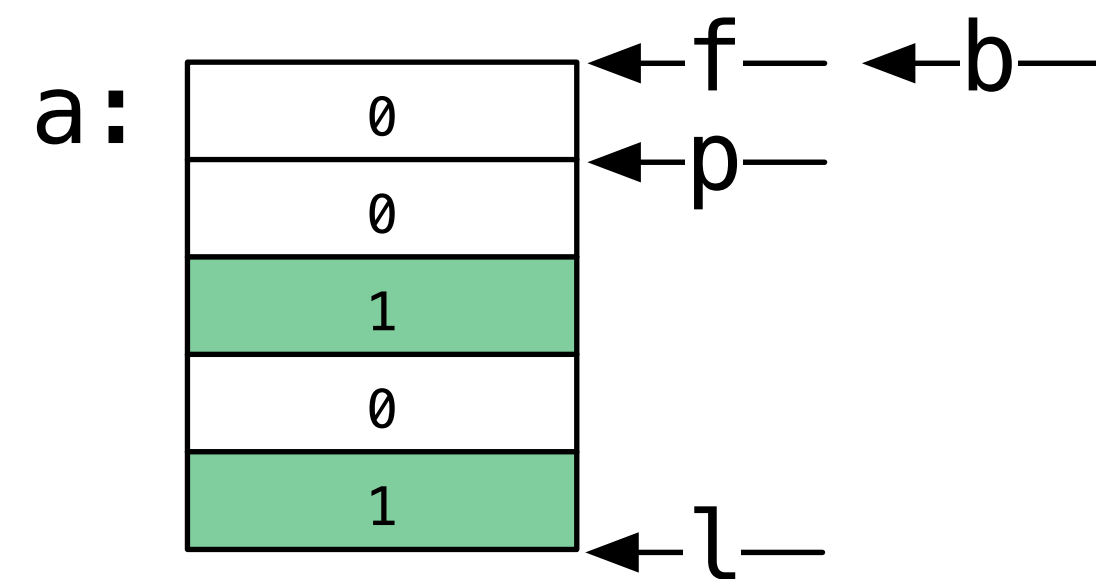


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```
template <forward_iterator I, class T>
auto remove(I f, I l, const T& a) -> I {
    auto b{find(f, l, a)};
    if (b == l) return b;
    auto p{next(b)};
    // invariant: `[f, b)` contain all the
```

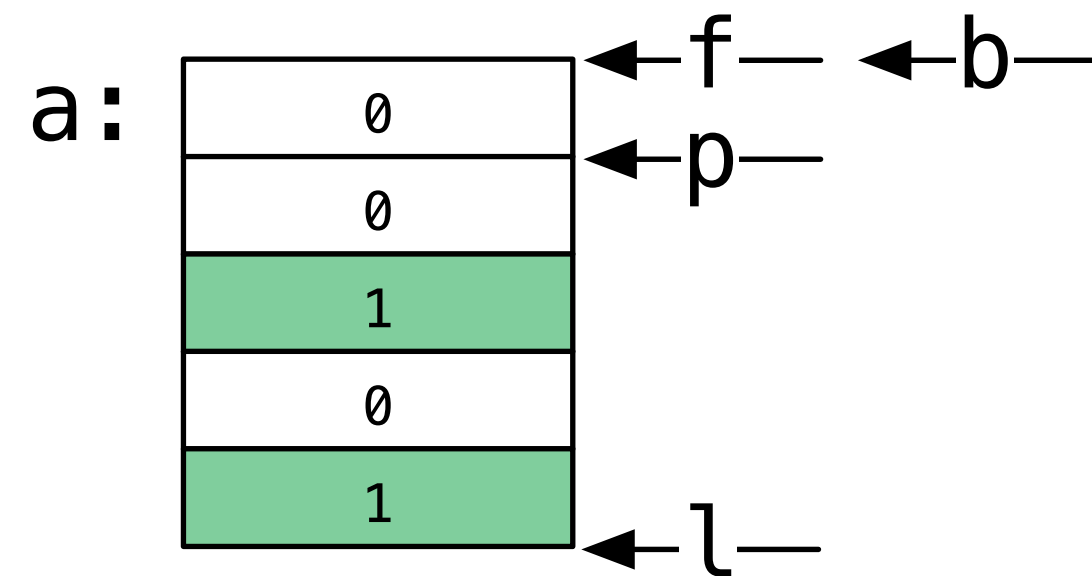


## Remove



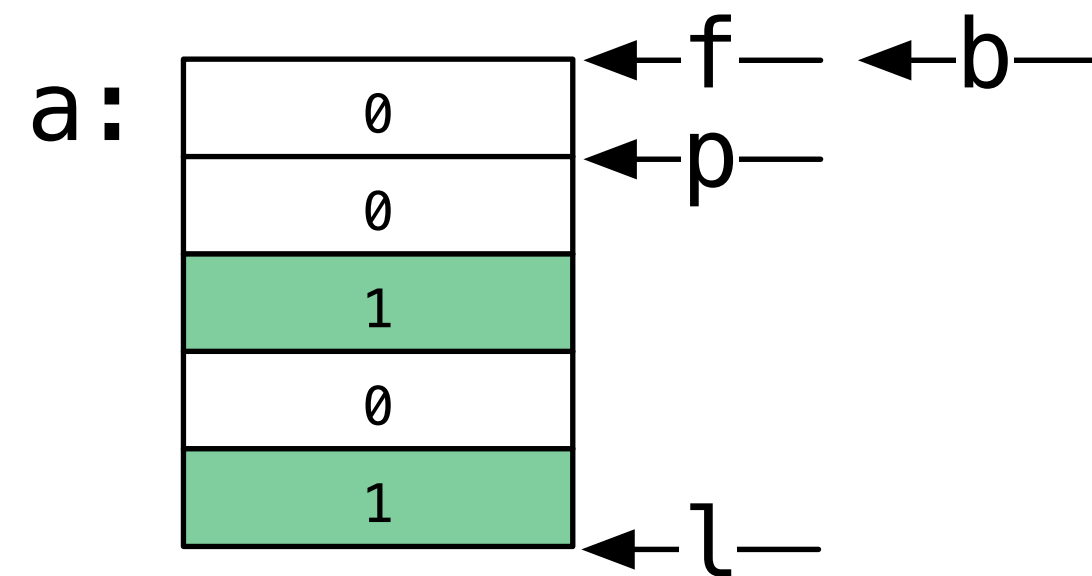
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}
```

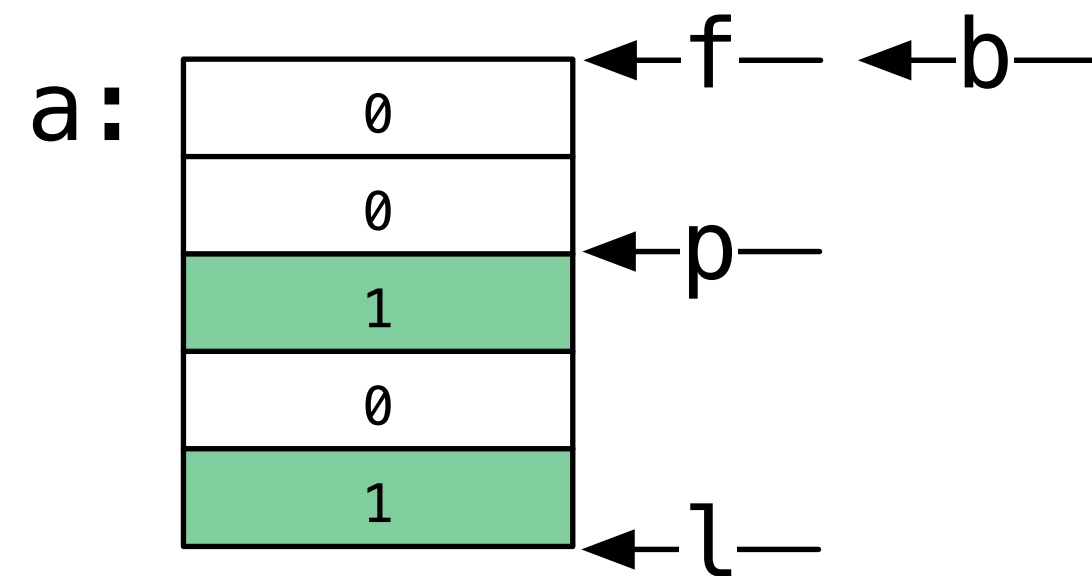
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            *b = std::move(*p);
            ++b;
        }
    }
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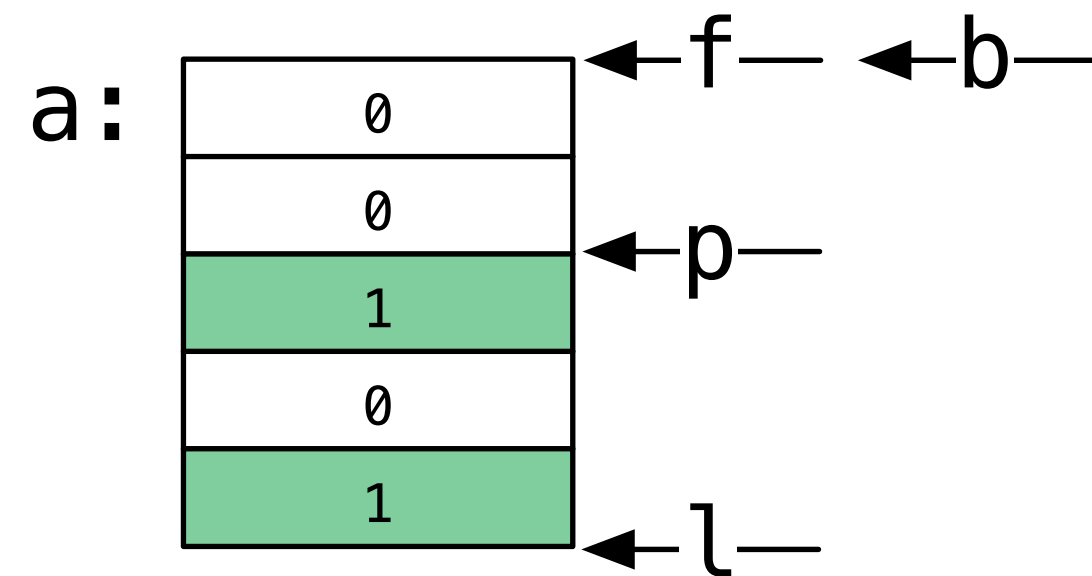


## Remove



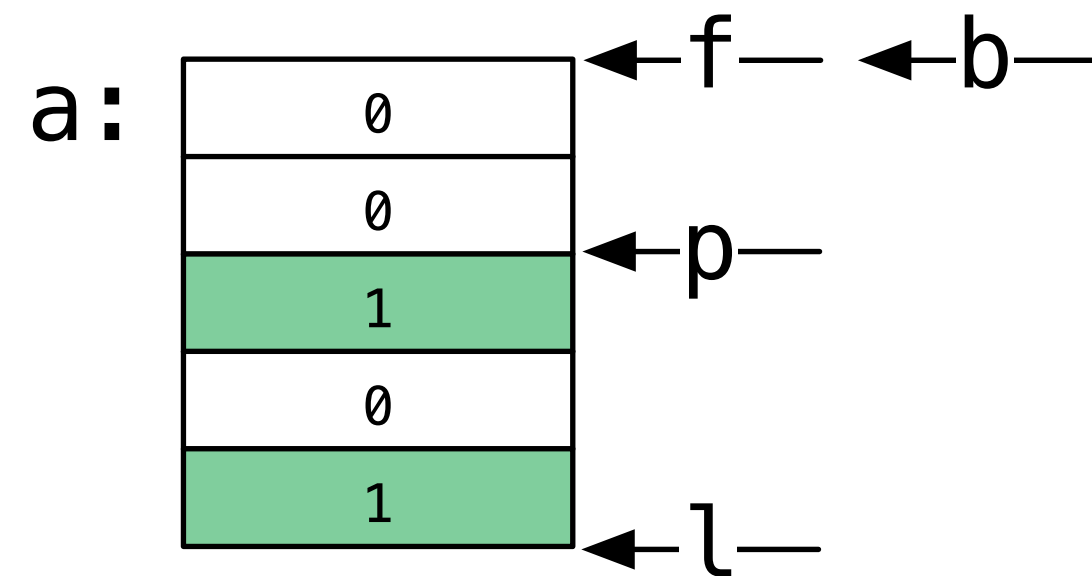
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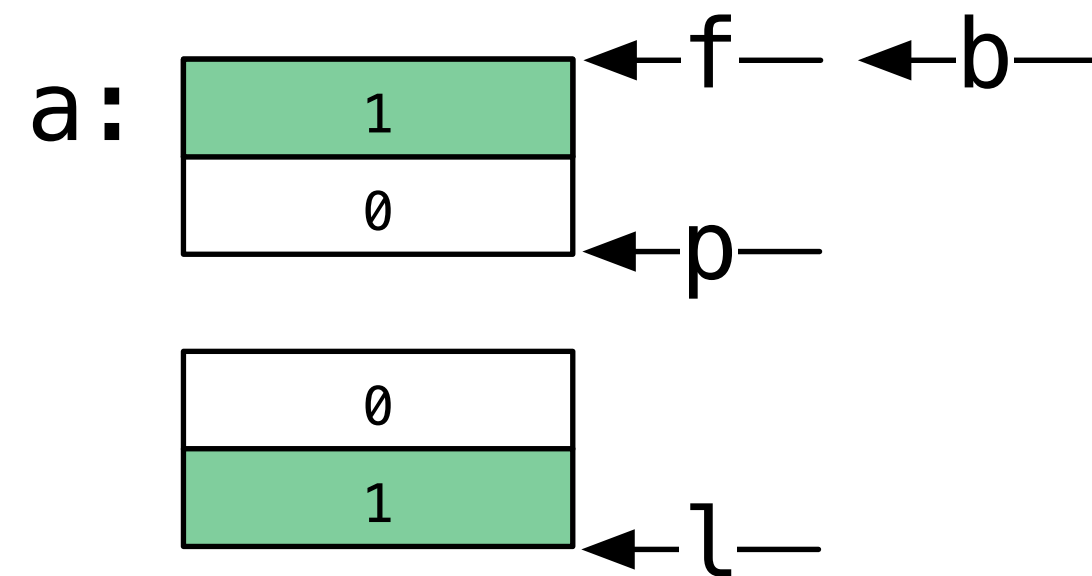
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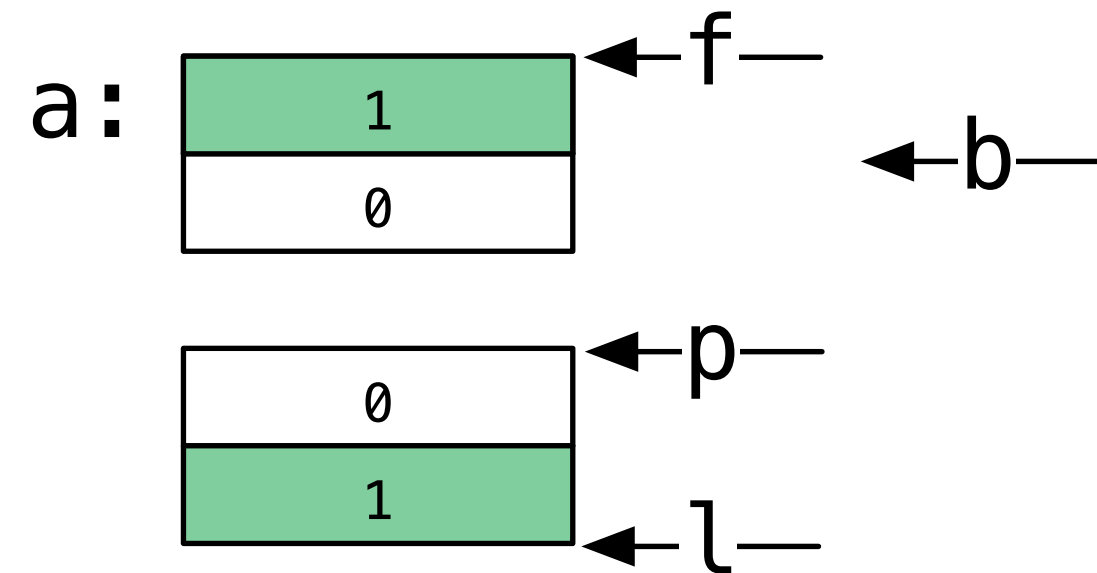
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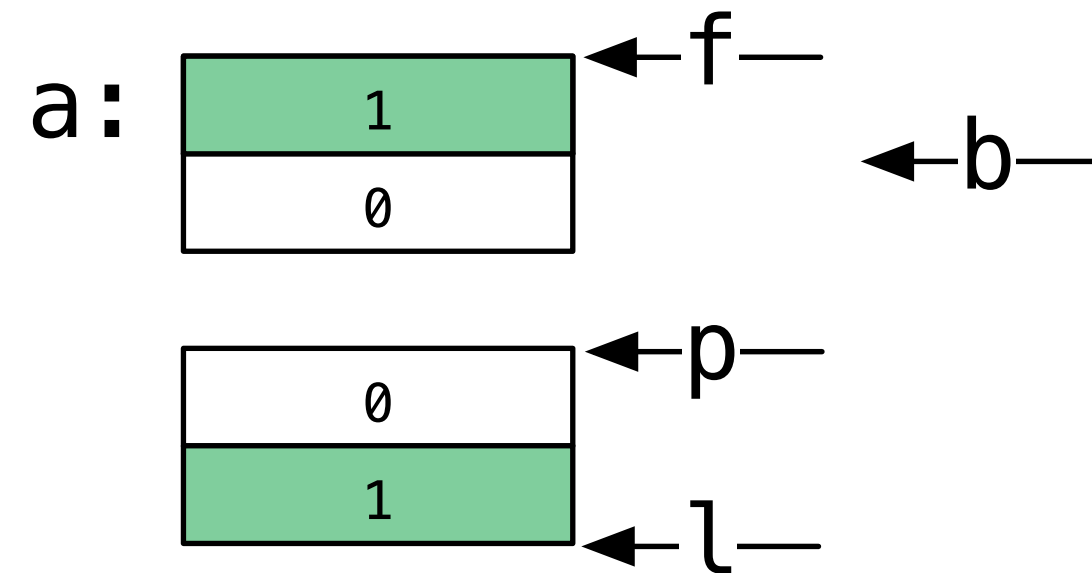
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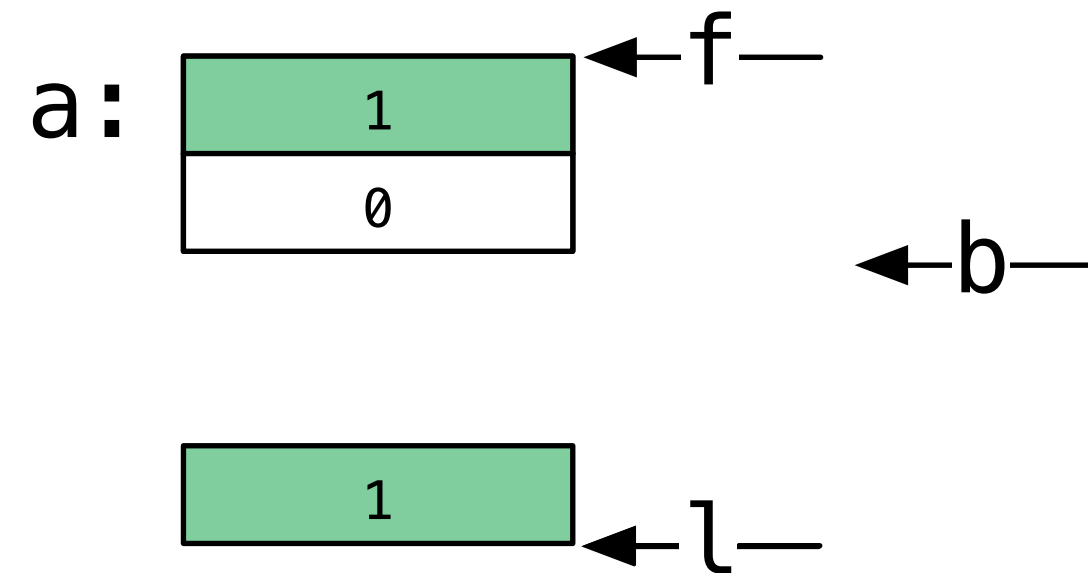
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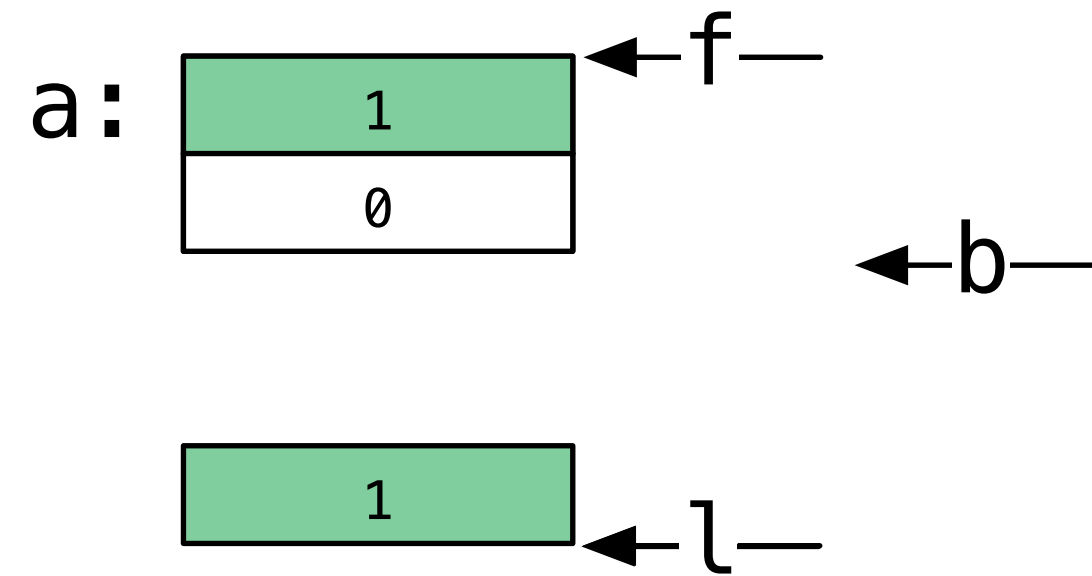
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Ways to represent a range of elements

- Closed interval  $[f, l]$
- Open interval  $(f, l)$
- Half-open interval  $[f, l)$ 
  - By strong convention, open on the right



# Half-Open Intervals

$[p, p)$  represents an empty range at position  $p$

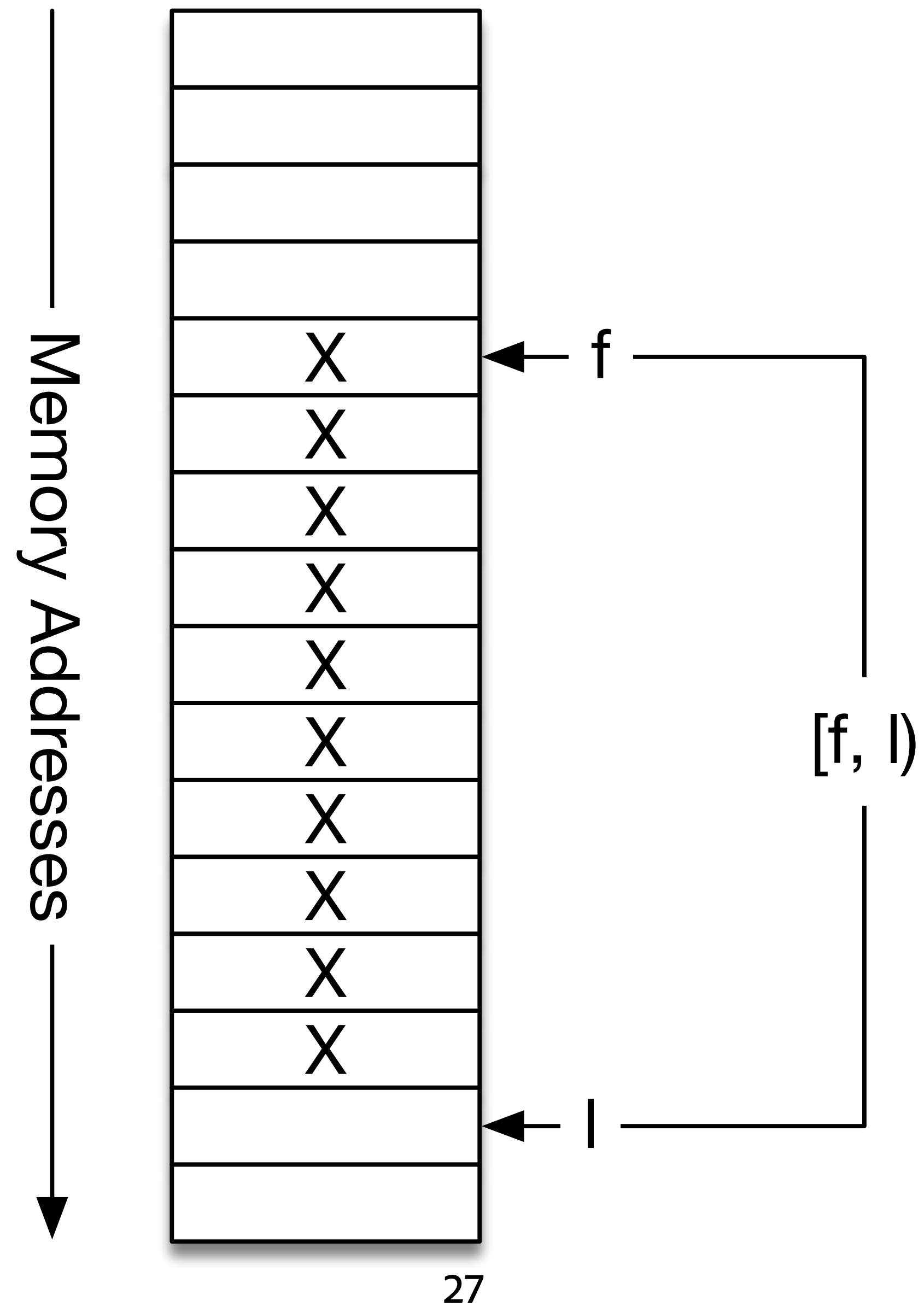
- All empty ranges are not equal

Cannot express the last item in a set with positions of the same set type

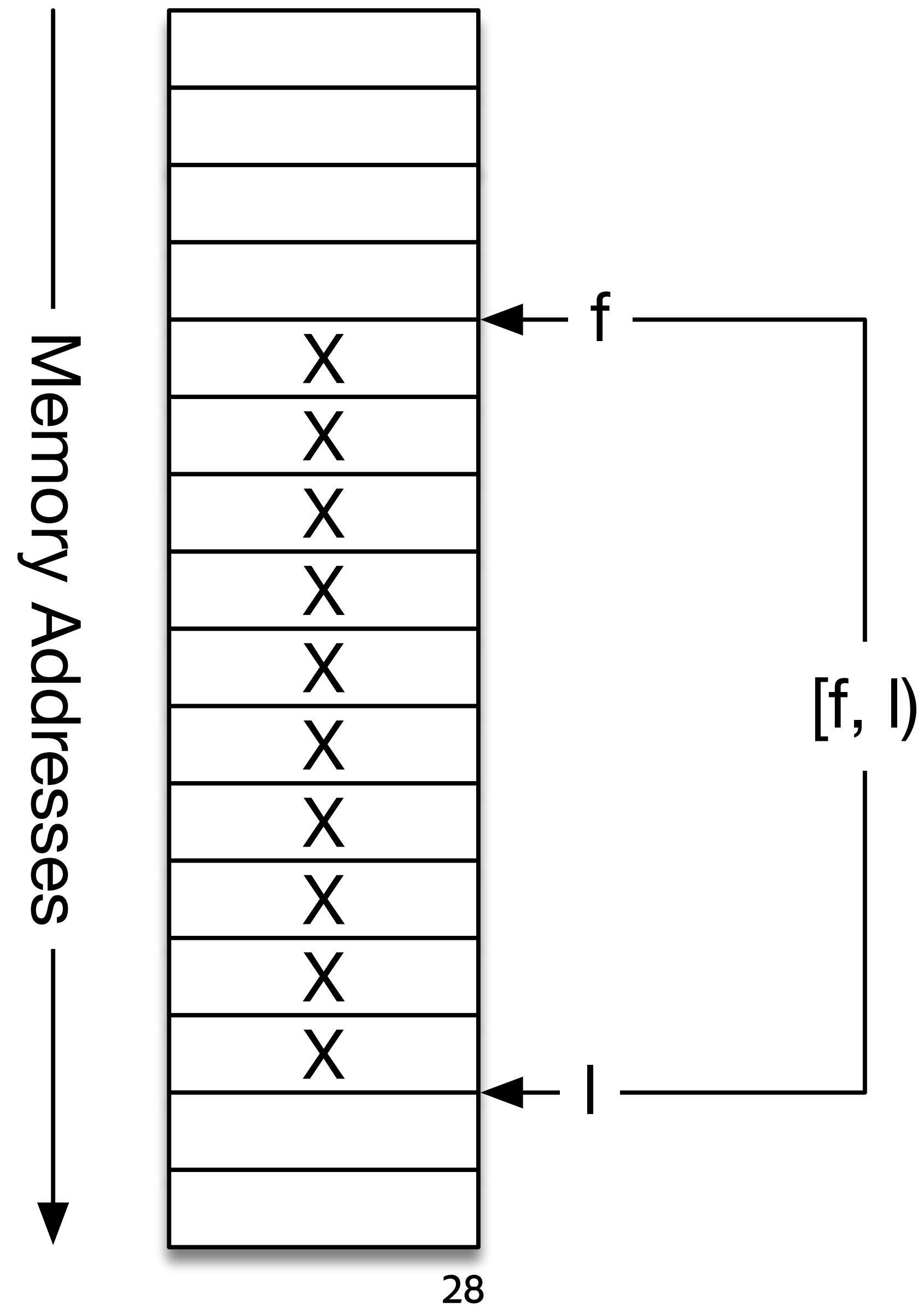
- i.e., `[INT_MIN, INT_MAX]` is not expressible as a half-open interval with type `int`

Think of the positions as the lines between the elements

# Half-Open Intervals



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In this model, there is a symmetry with reverse ranges  $(l, f]$

- The dereference operation is asymmetric. dereferencing at a position  $p$  is the value in  $[p, p + 1)$

Half-open intervals avoid off-by-one errors and confusion about *before* or *after*

In C and C++, half-open intervals are built into the language. For any object,  $a$ ,  $\&a$  is a pointer to the object, and  $\&a + 1$  is a valid pointer but may not be dereferenceable.

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```
int a{42};
```

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- Any object can be treated as a range of one element

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copy(&a, &a + 1, ostream_iterator<int>(cout));
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In this model, there is a symmetry with reverse ranges  $(l, f]$

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# Much More

Composing Algorithms

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∅ Patents

# Q & A



## About the artist

### Dan Zucco

London-based 3D art and motion director Dan Zucco creates repeating 2D patterns and brings them to life as 3D animated loops. Inspired by architecture, music, modern art, and generative design, he often starts in Adobe Illustrator and builds his animations using Adobe After Effects and Cinema 4D. Zucco's objective for this piece was to create a geometric design that felt like it could have an infinite number of arrangements.

Made with

 Adobe Illustrator

 Adobe After Effects



Adobe

Bē

Artwork by Dan Zucco