



Algorithms

Rubric: No Raw Loops

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Definition

“An *Algorithm* is a process or set of rules to be followed in calculations or other problem-solving operations, especially by a computer.” – *New Oxford American Dictionary*

A Simple Algorithm

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- What does this line of code do?

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// r is the minimum of `a` and `b`  
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A Simple Algorithm

```
int r = min(a, b);
```

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/// returns the minimum of `a` and `b`  
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 - Or implied by the preconditions of the algorithm
- The postconditions for the algorithm must follow from the sequence of statements

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Functions allow us to build a vocabulary focused on semantics.

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 - copular constructions: `is_blue`
 - consider a verb if the complexity is greater than expected

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 - `name()` not `get_name()`

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 - Prefer sink argument and result to in-out arguments
- spans, views, iterator pairs, and so on are a way to pass a range of objects as if they were a simple argument. The `value_type` of the range determines if it is a *let* (const) argument or *in-out* (not const), and input ranges are used for *sink* arguments

Argument Types

```
void display(const vector<unique_ptr<widget>>& a) {  
    //...  
    a[0]->set_name("displayed"); // DONT  
    //...  
}
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- Meaning value
 - A meaningless object should not be passed as an argument (i.e., an invalid pointer).

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vector a{0, 0, 1, 0, 1 };  
erase(a, a[0]);  
display(a);
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{ 1, 0 }
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erase(a, copy(a[0]));  
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container<int> a{ 0, 1, 2, 3 };  
auto f = begin(a);  
a.push_back(5);  
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- Example: the reference returned from `vector::back()`

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 - Examples: `swap()`, `exchange()`, `min()`, `max()`, `clamp()`, `tolower()`...
- A *non-trivial* algorithm requires iteration
 - iteration may be implemented as a loop or recursion

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 - An invariant that holds at the start of the iteration and after each step
 - A finite decreasing property where termination happens when the property is zero
- The postcondition of the iteration is the above invariant when the decreasing property reaches zero

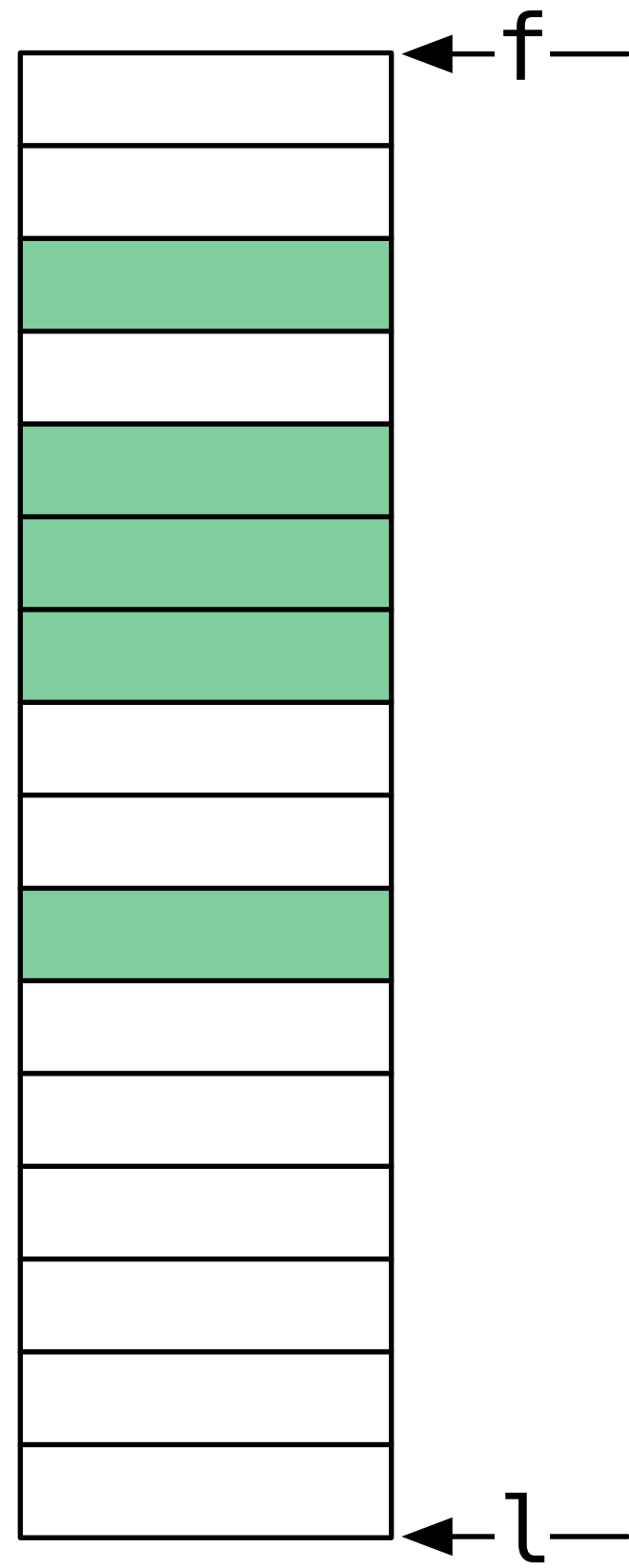
Remove

Remove

```
/**  
    Removes values equal to `a` in the range `[f, l)`.  
  
    \return the position, `b`, such that `[f, b)` contains all the  
            values in `[f, l)` not equal to `a`  
            values in `[b, l)` are unspecified  
*/
```

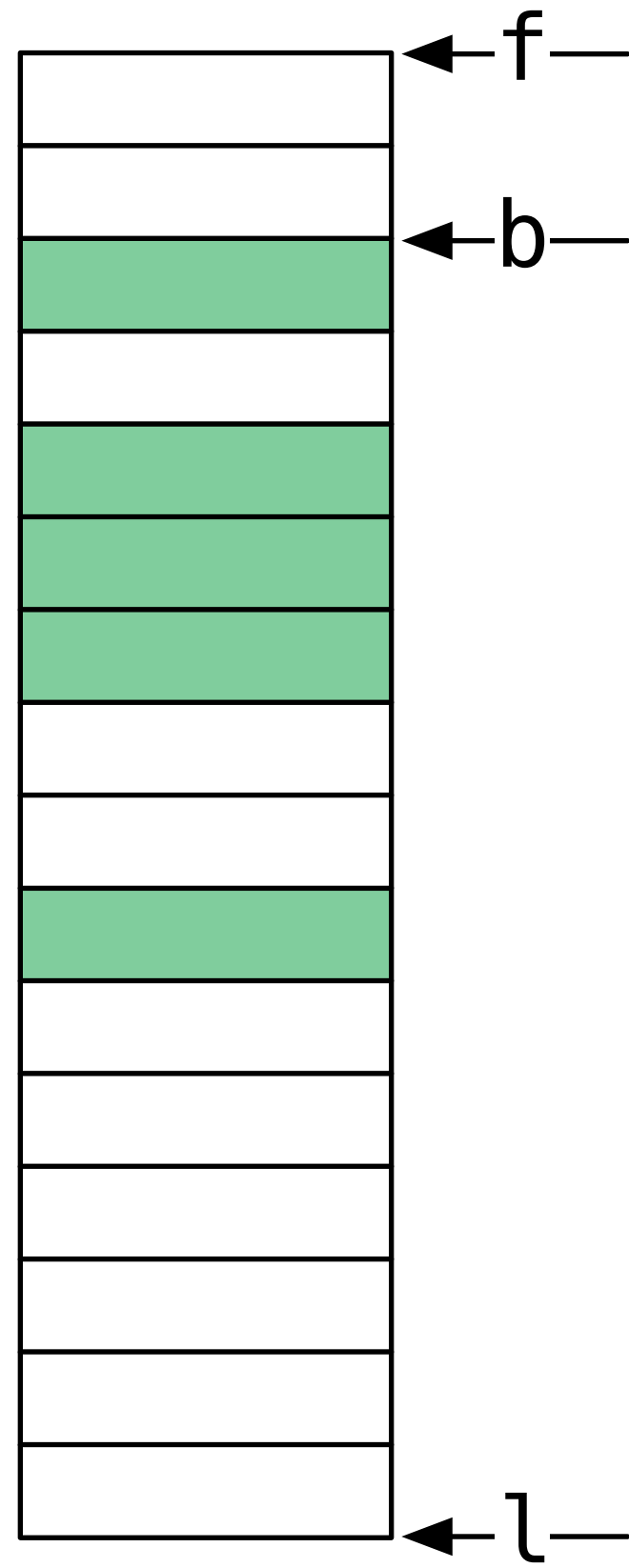
```
template <std::forward_iterator I, class T>  
auto remove(I f, I l, const T& a) -> I;
```

Remove



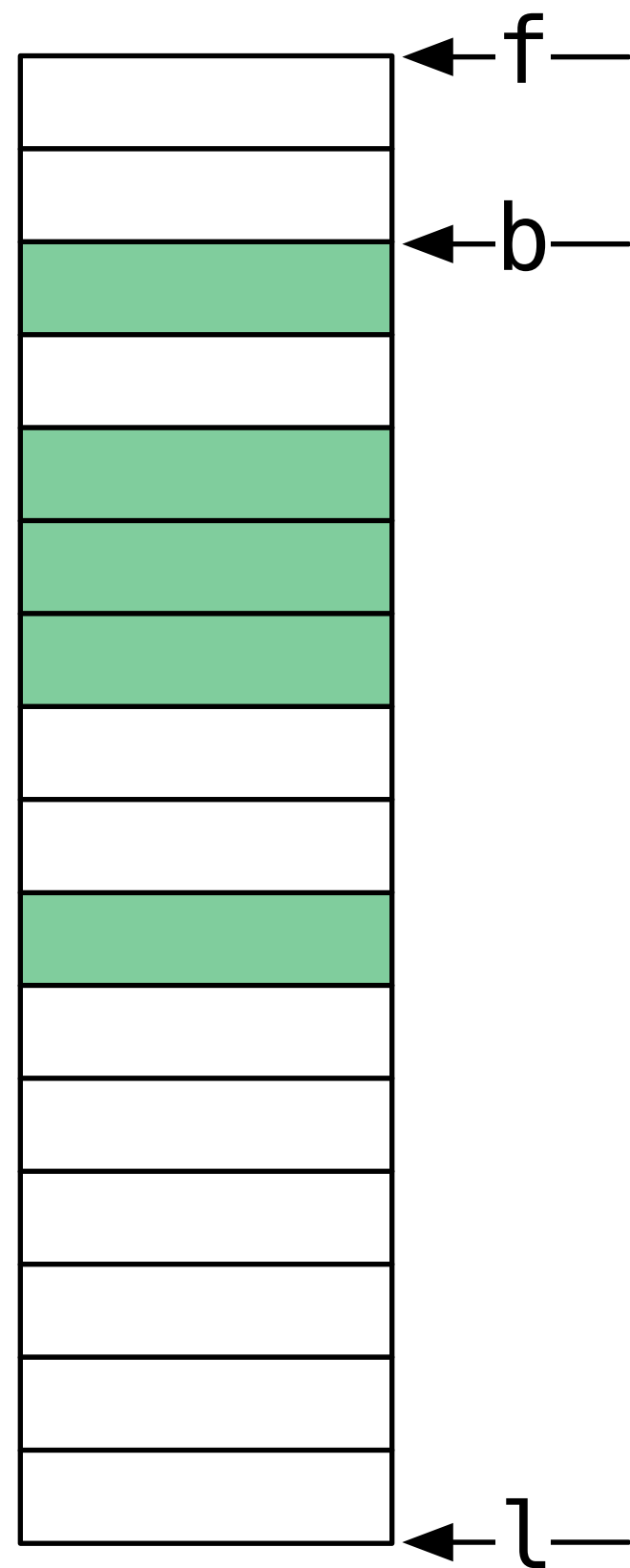
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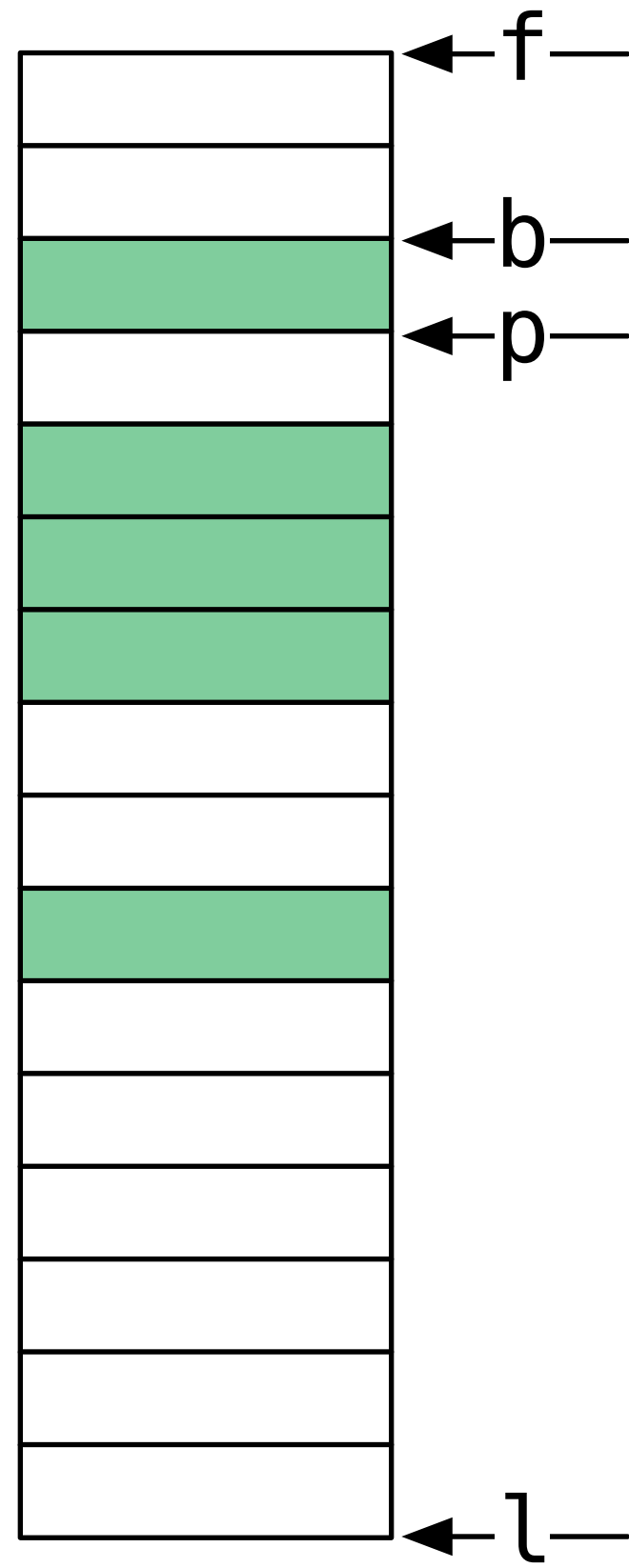
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auto remove(I f, I l, const T& a) -> I {
    auto b{find(f, l, a)};
}
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Remove



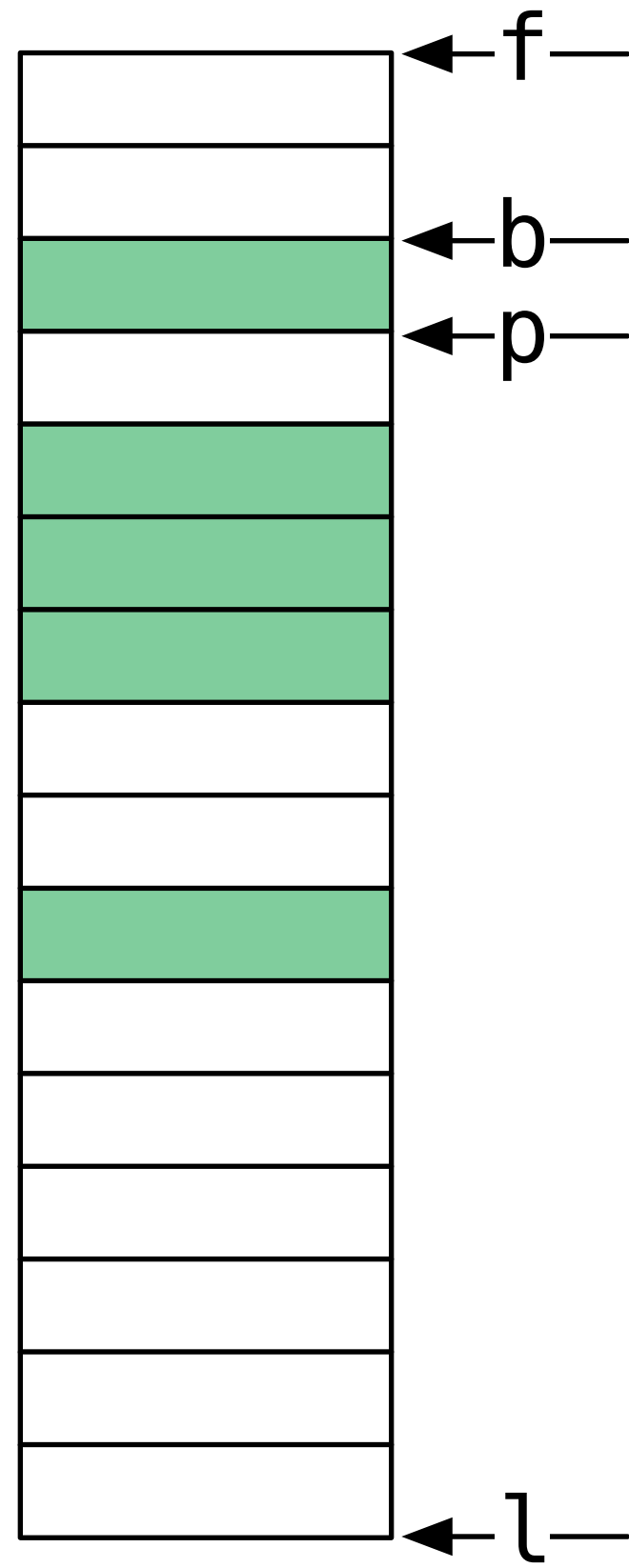
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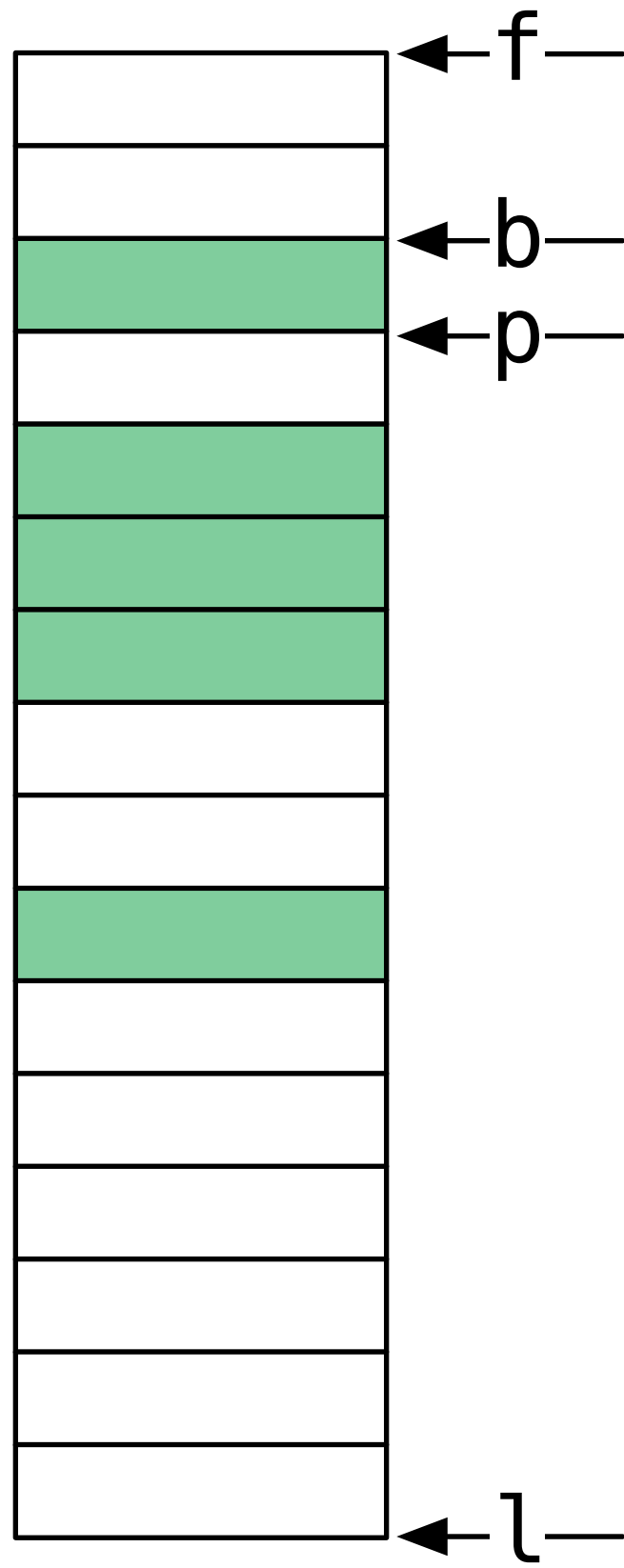
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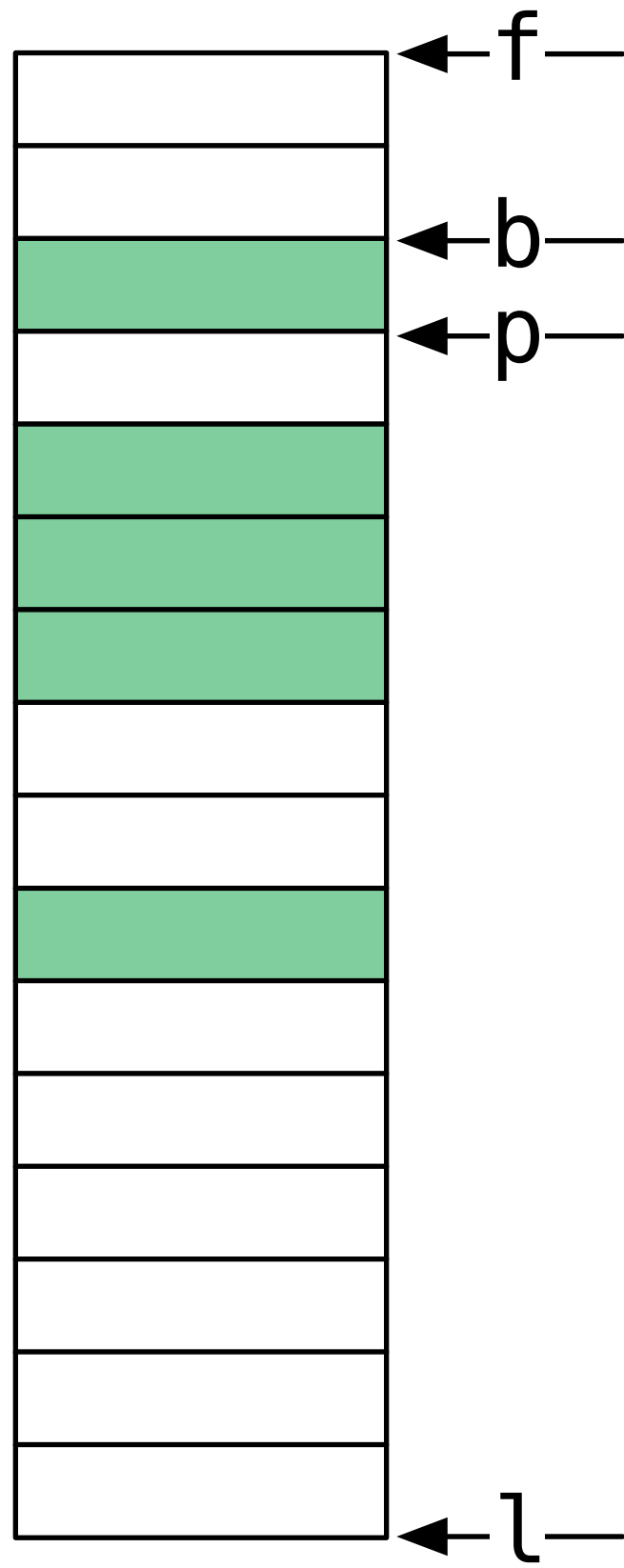
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    // invariant: `[f, b)` contain all the
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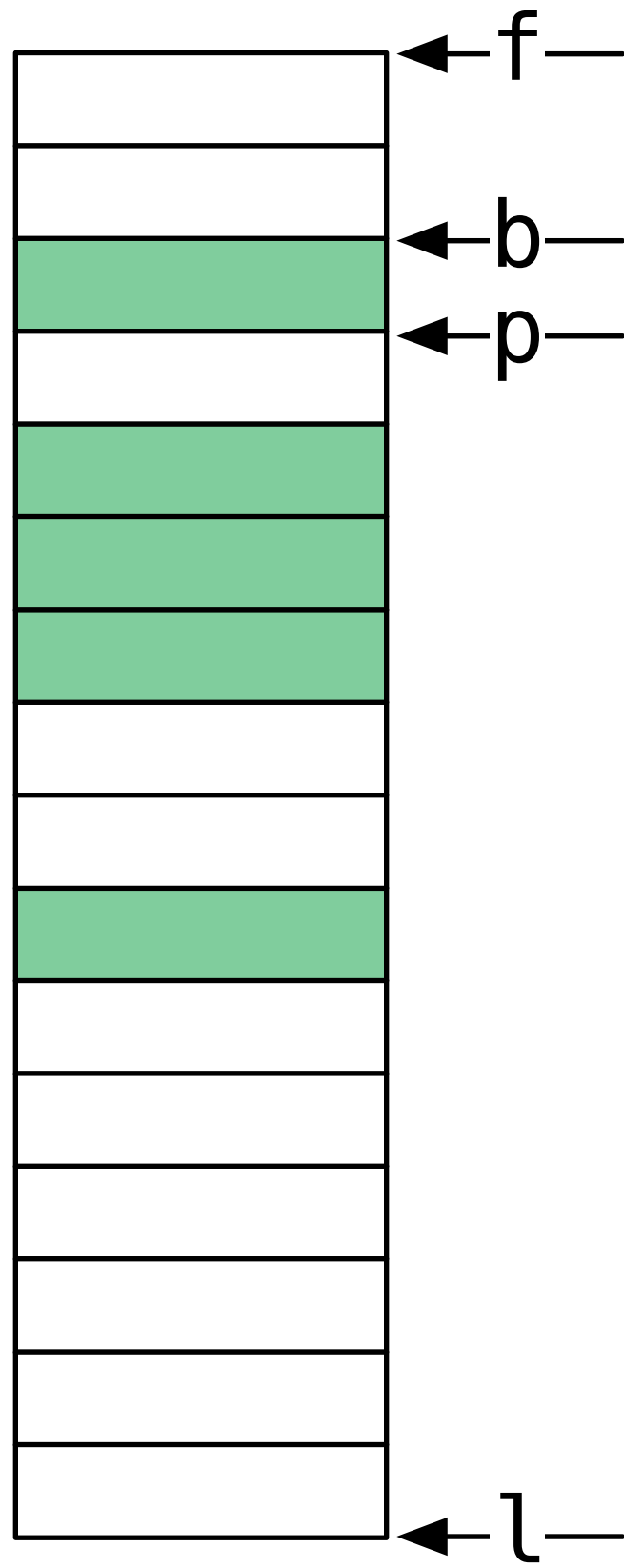
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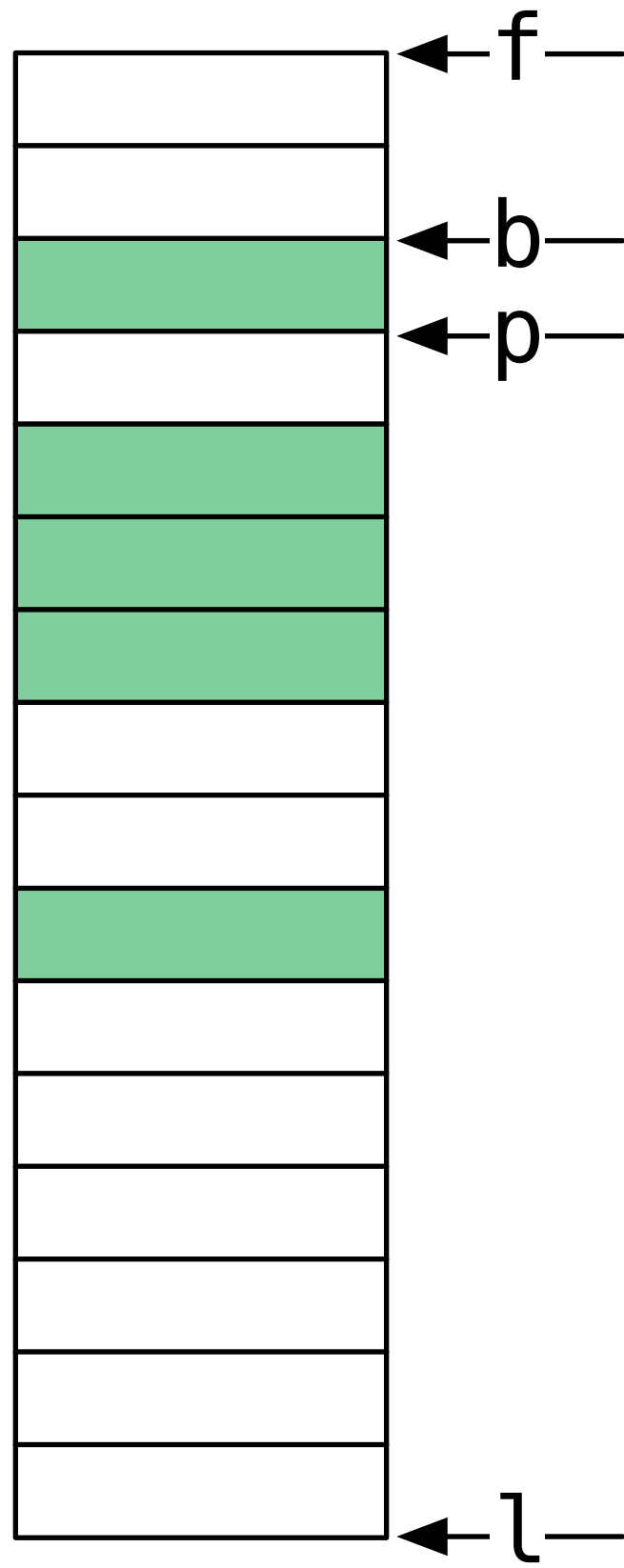
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    // decreasing: `distance(p, l)`
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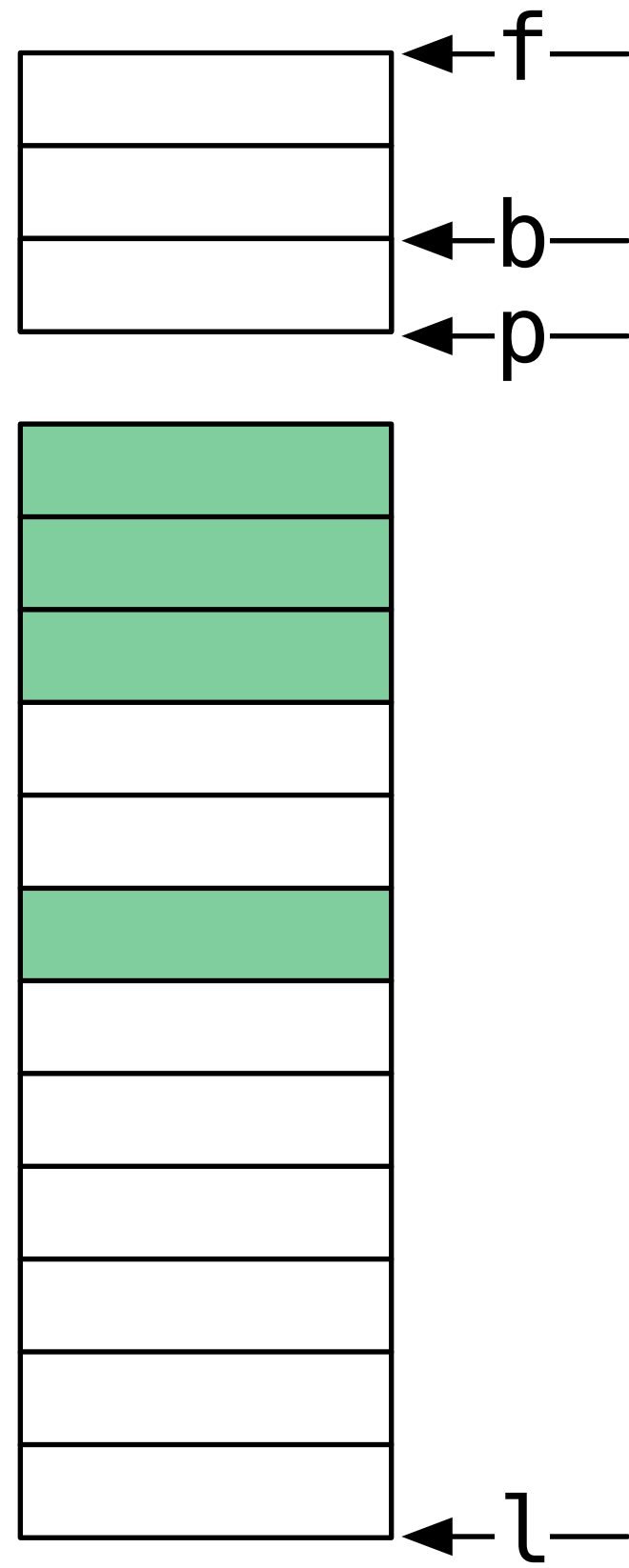
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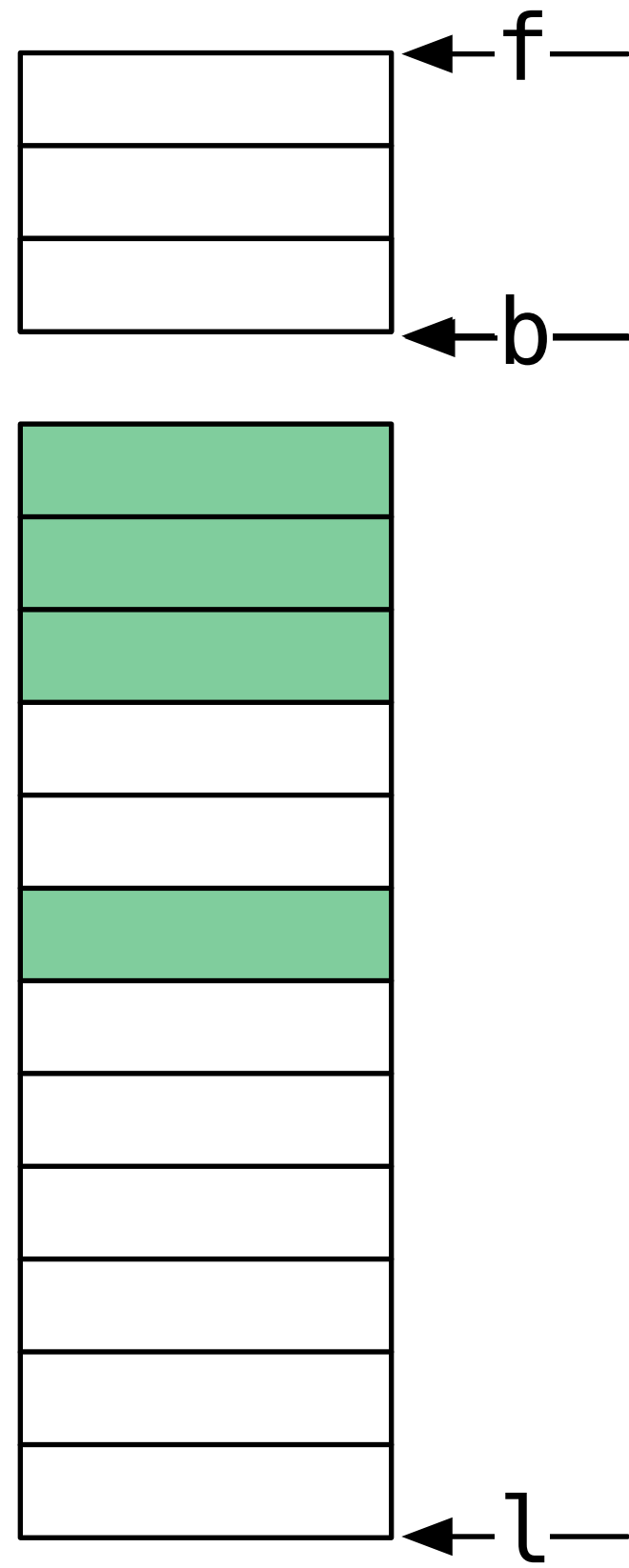
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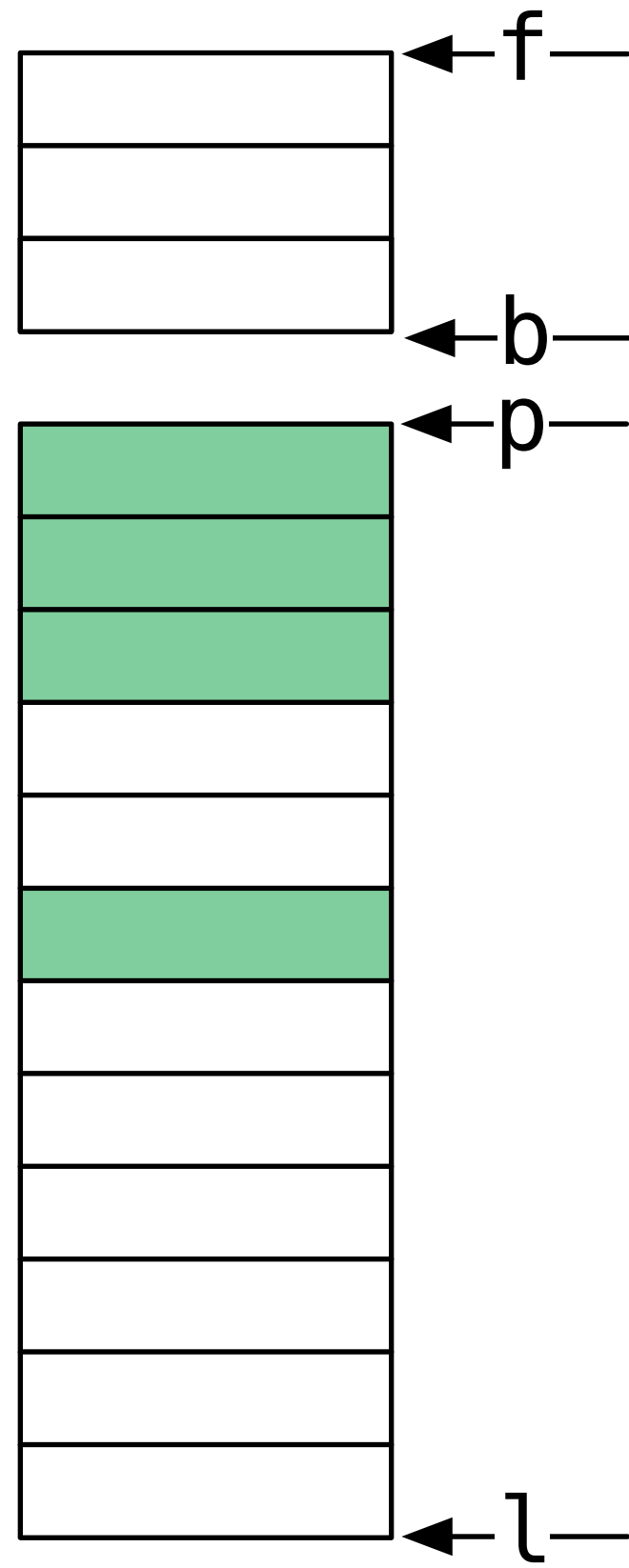
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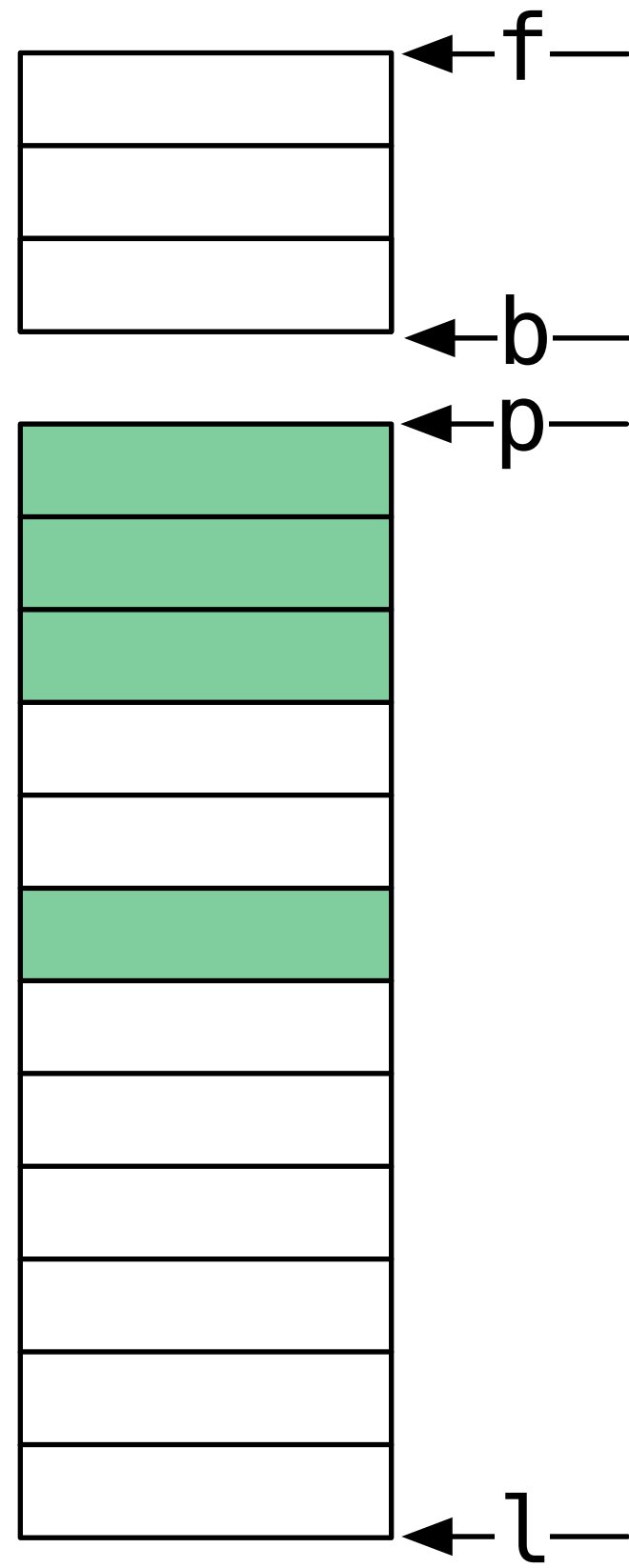
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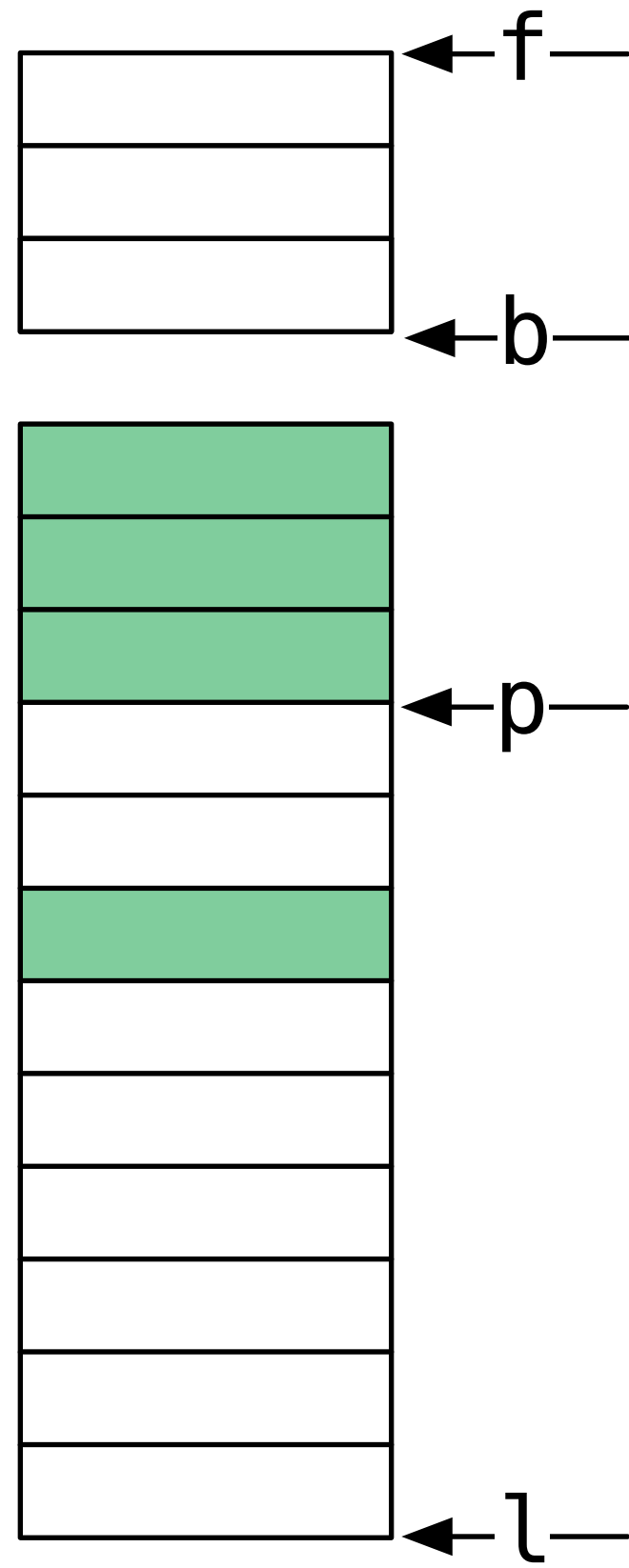
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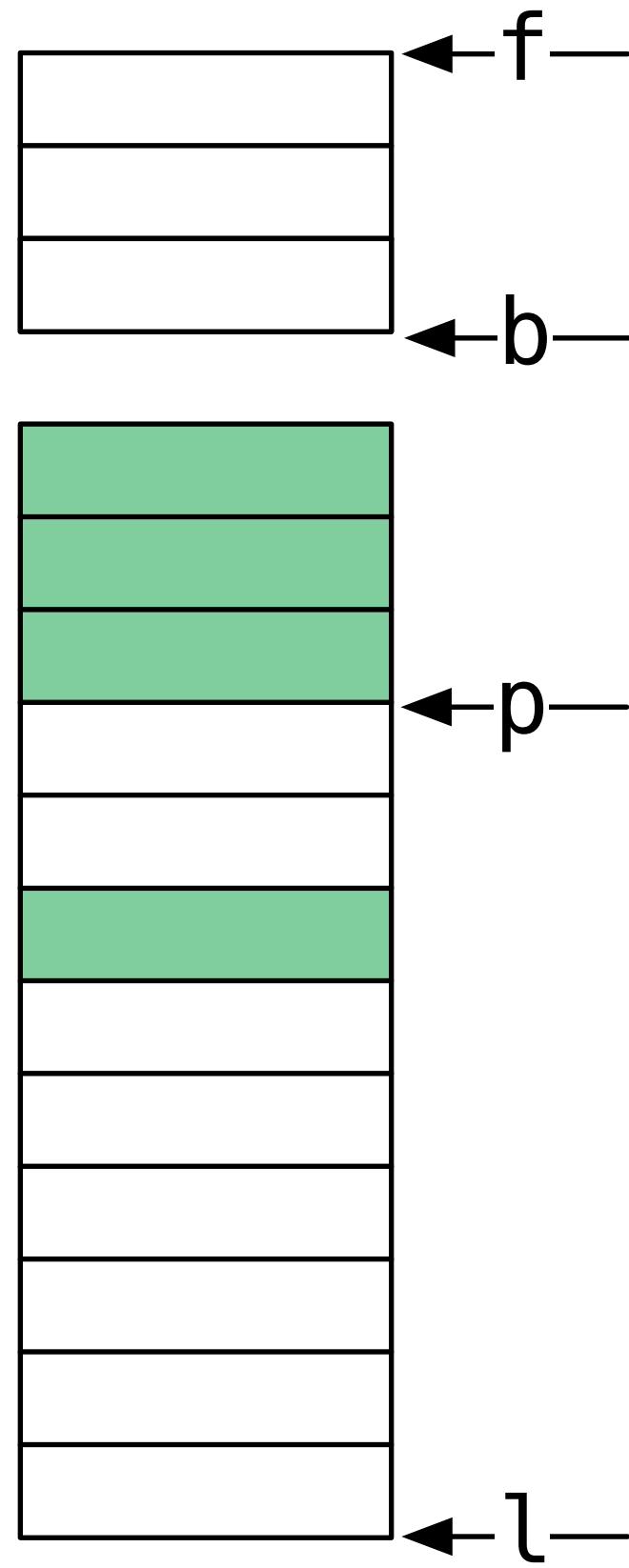
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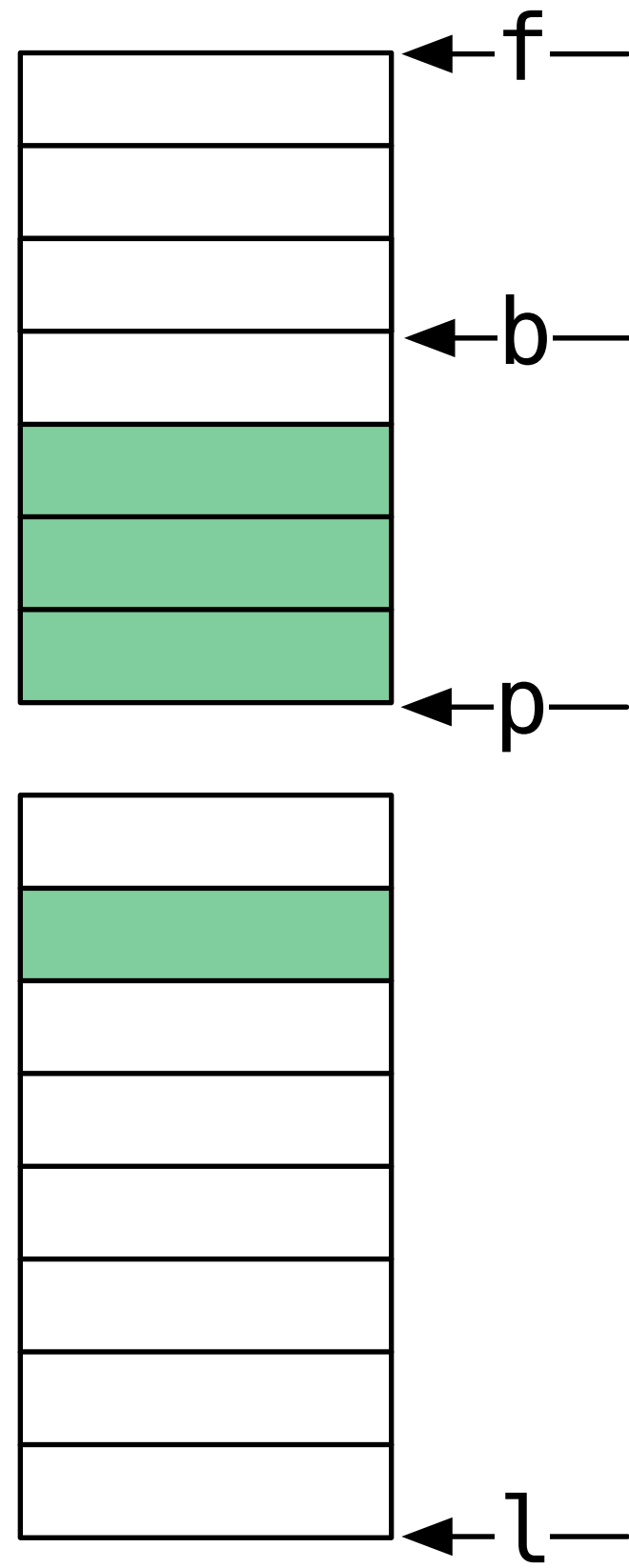
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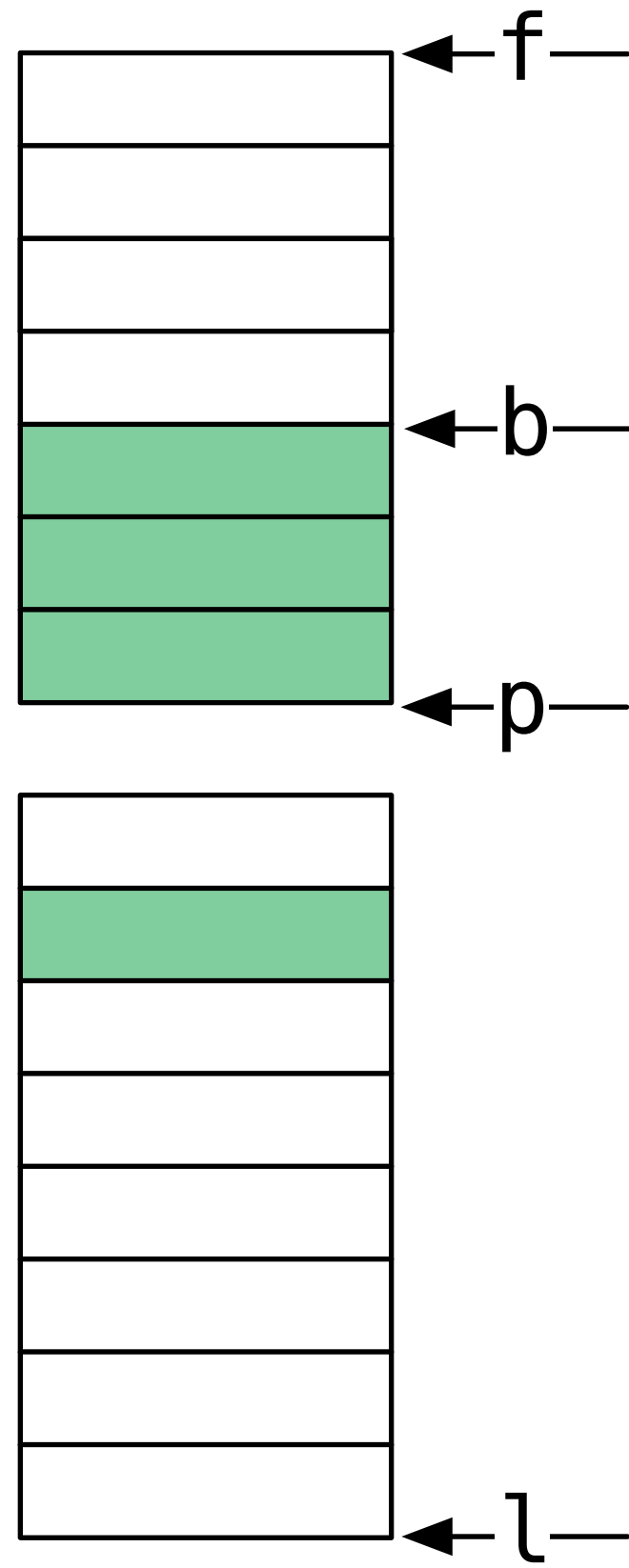
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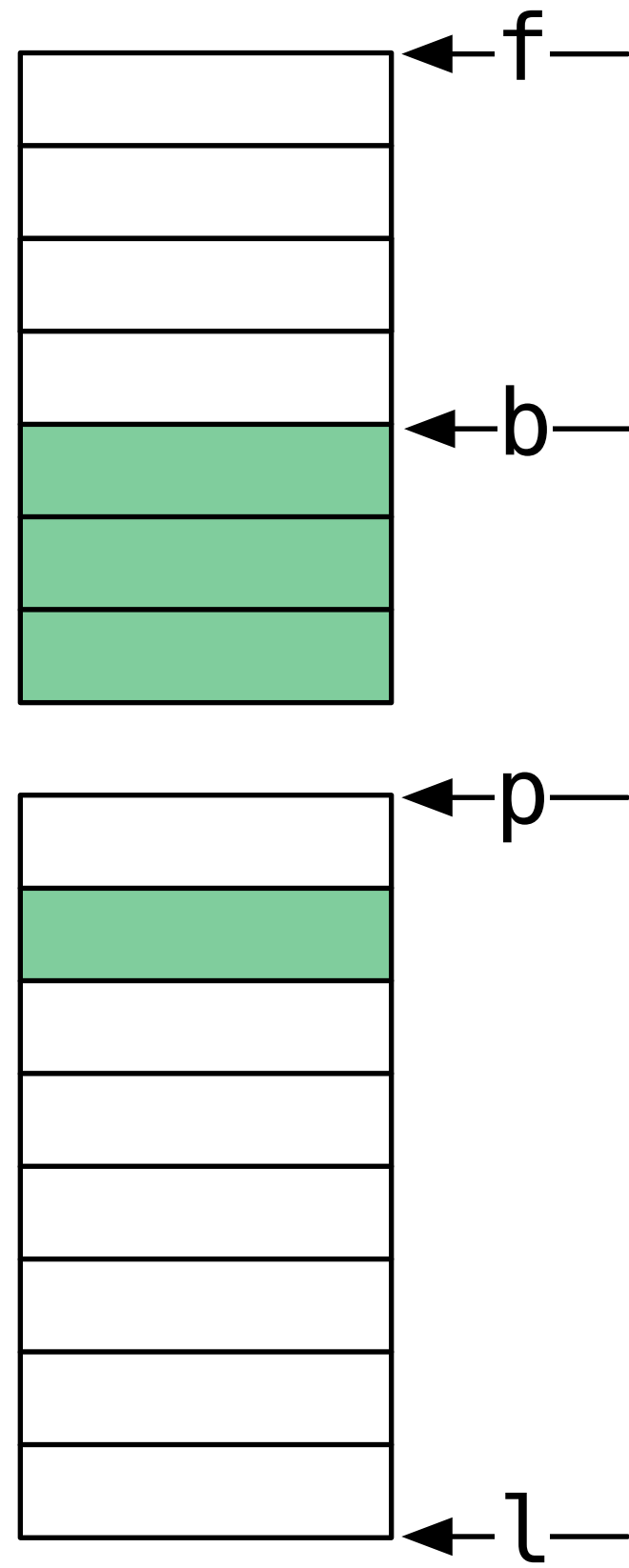
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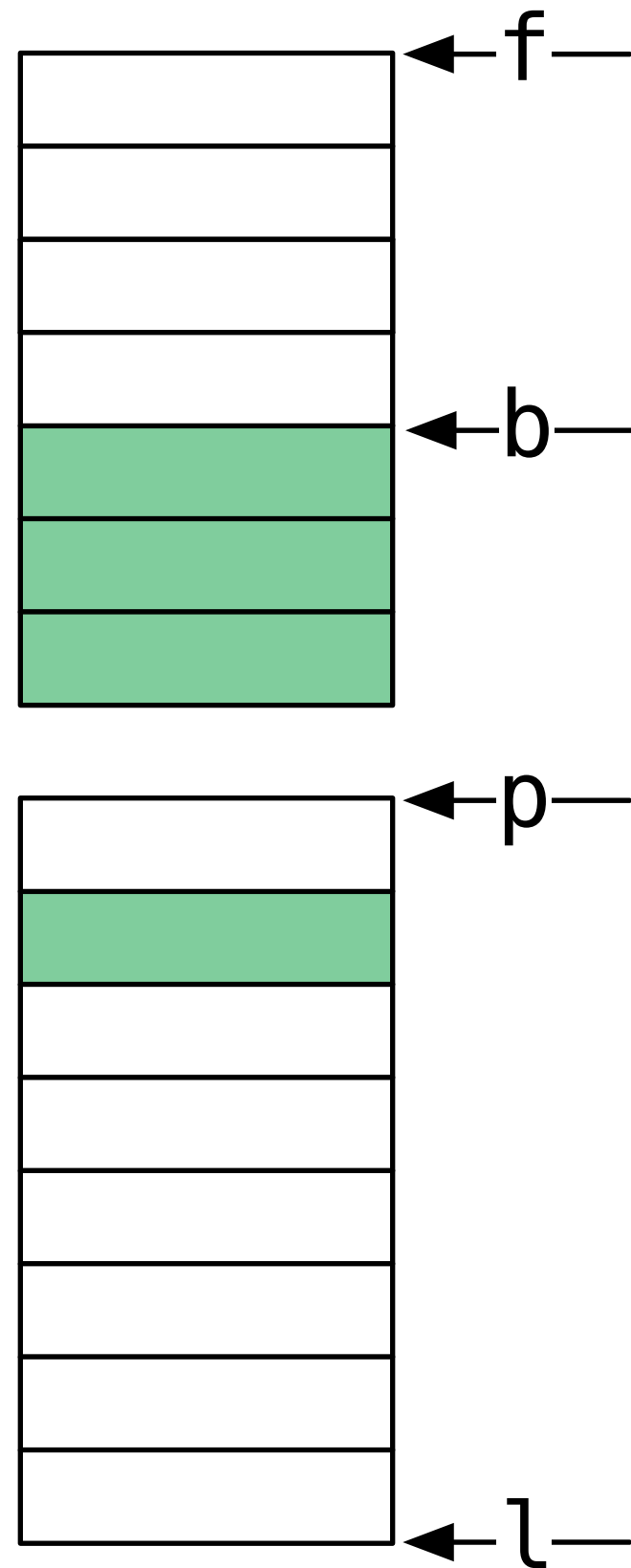
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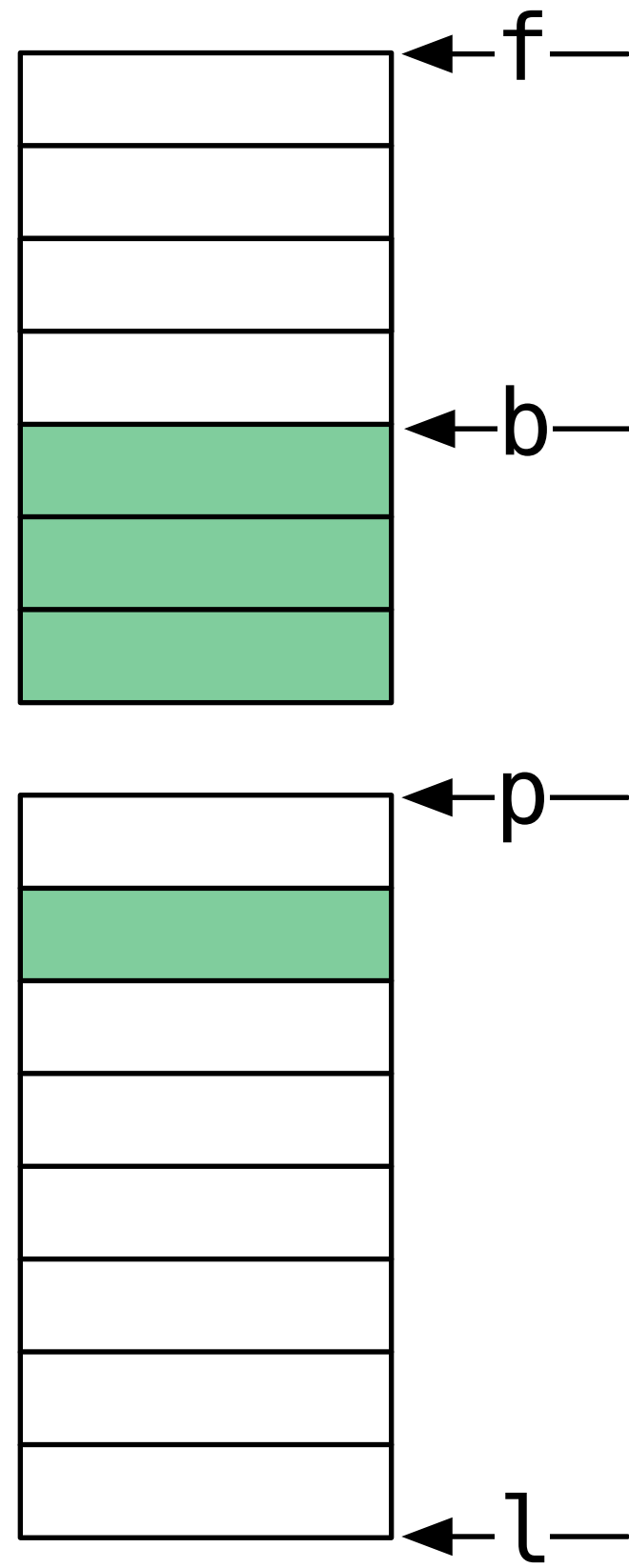
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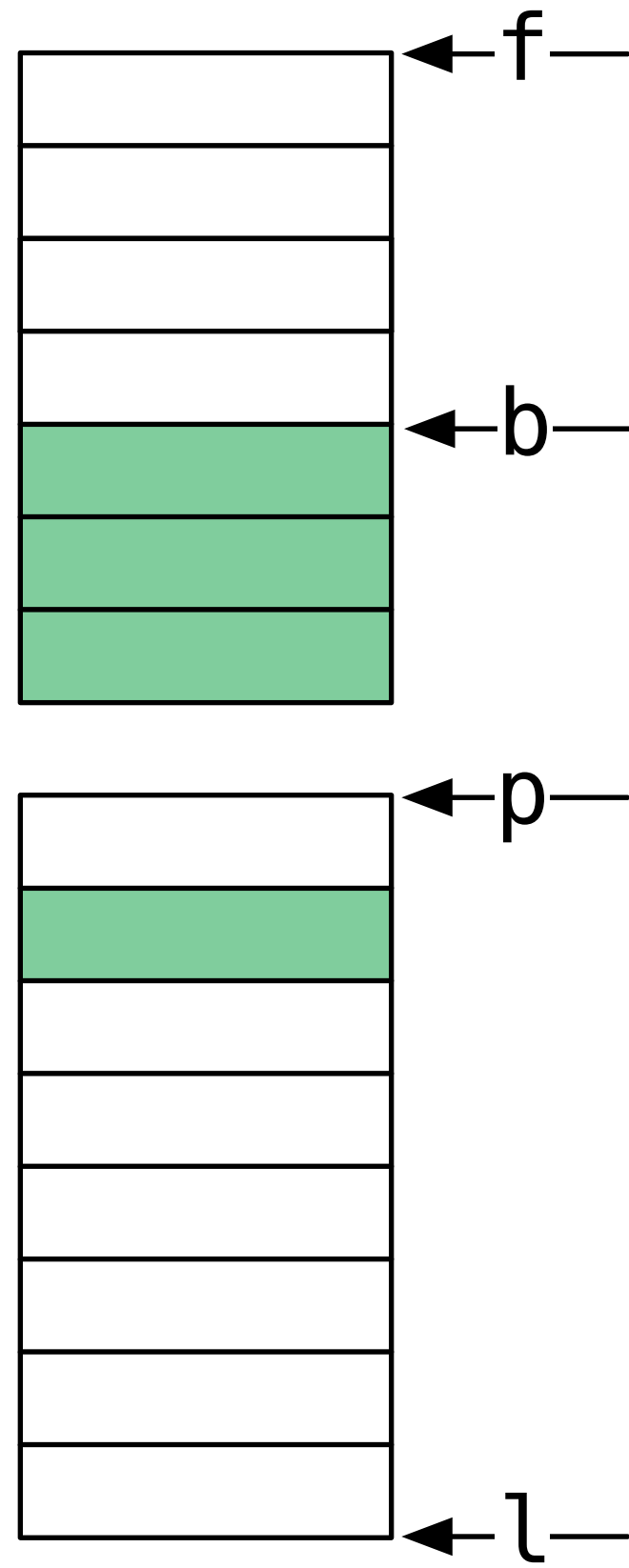
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    // values in `[f, p)` not equal to `a`
    // decreasing: `distance(p, l)`
    while (p != l) {
        if (*p != a) {
            *b = std::move(*p);
            ++b;
        }
        ++p;
    }
}
```


Remove



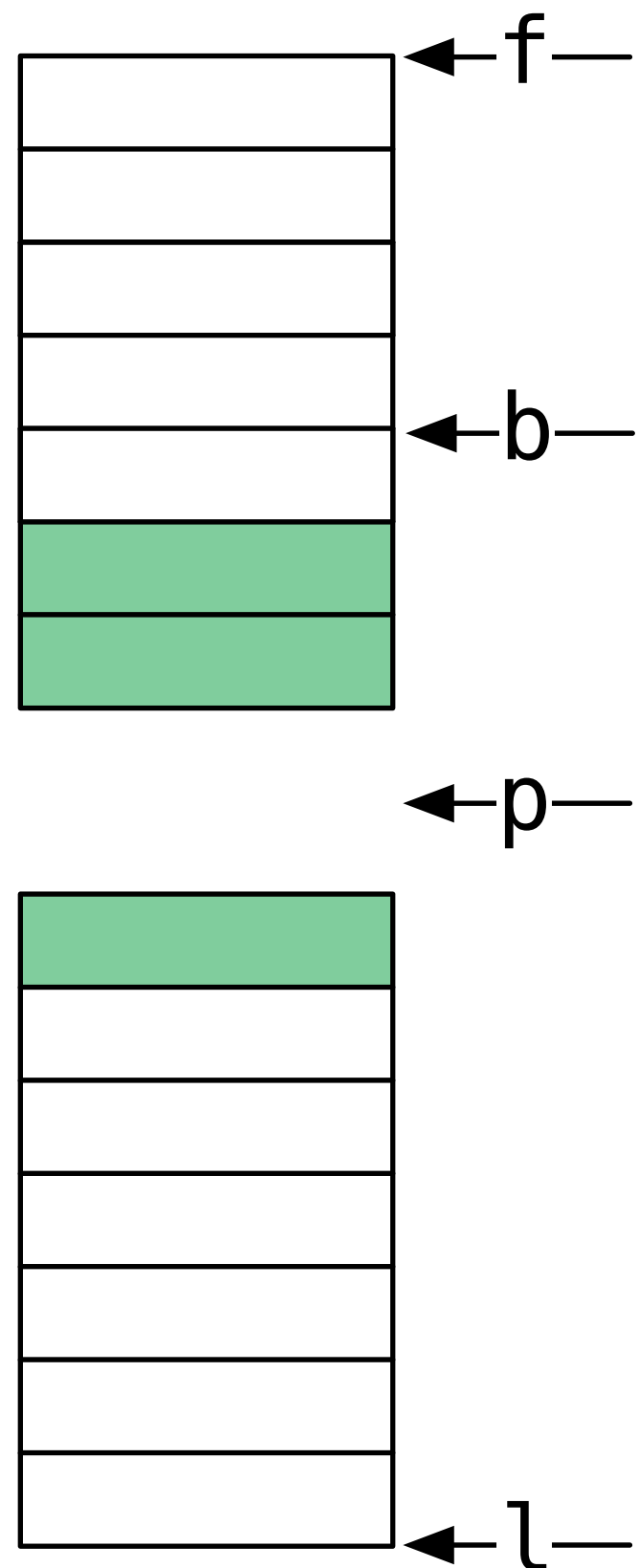
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template <std::forward_iterator I, class T>
auto remove(I f, I l, const T& a) -> I {
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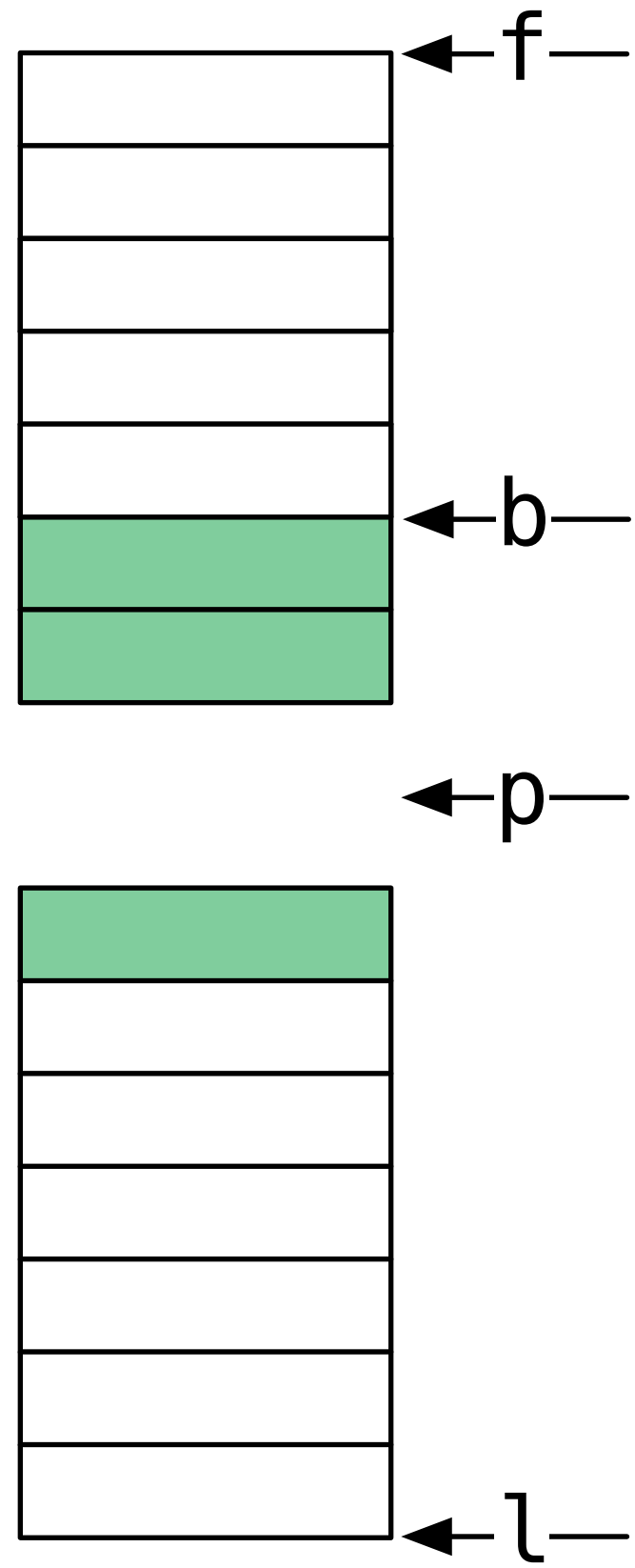
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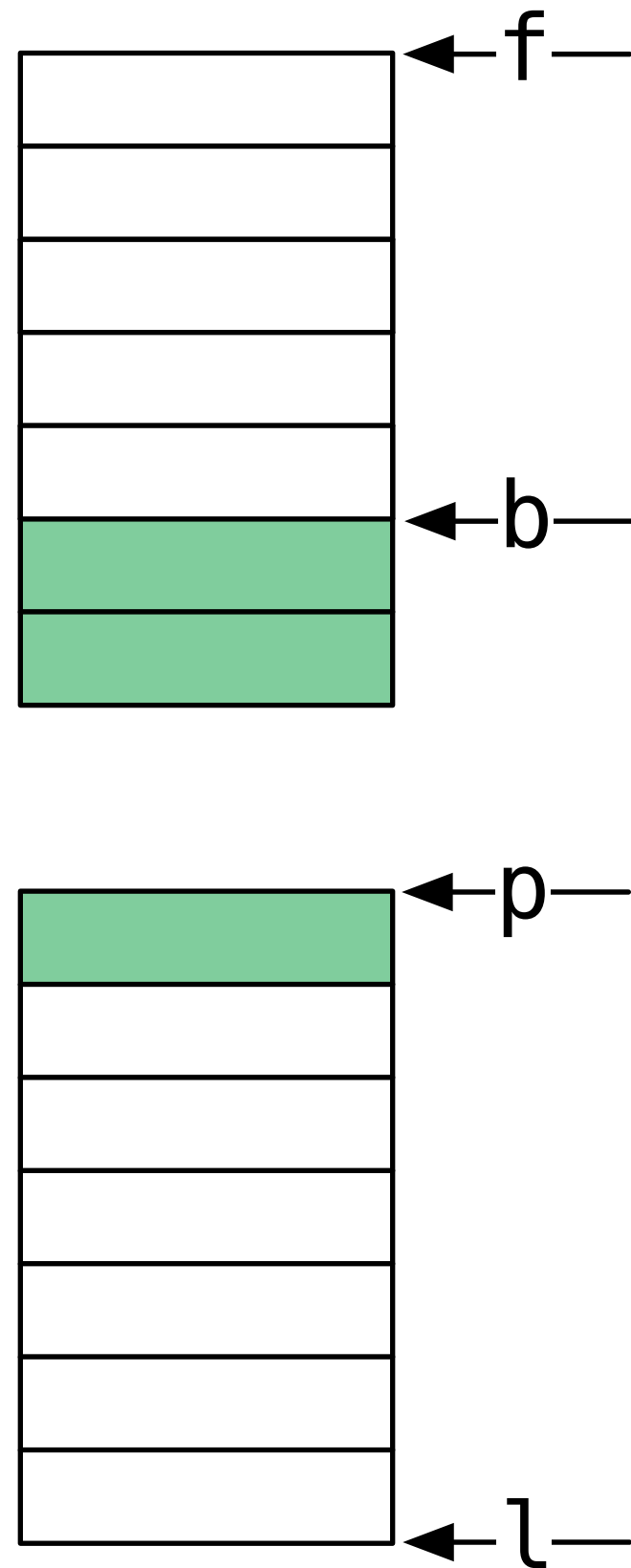
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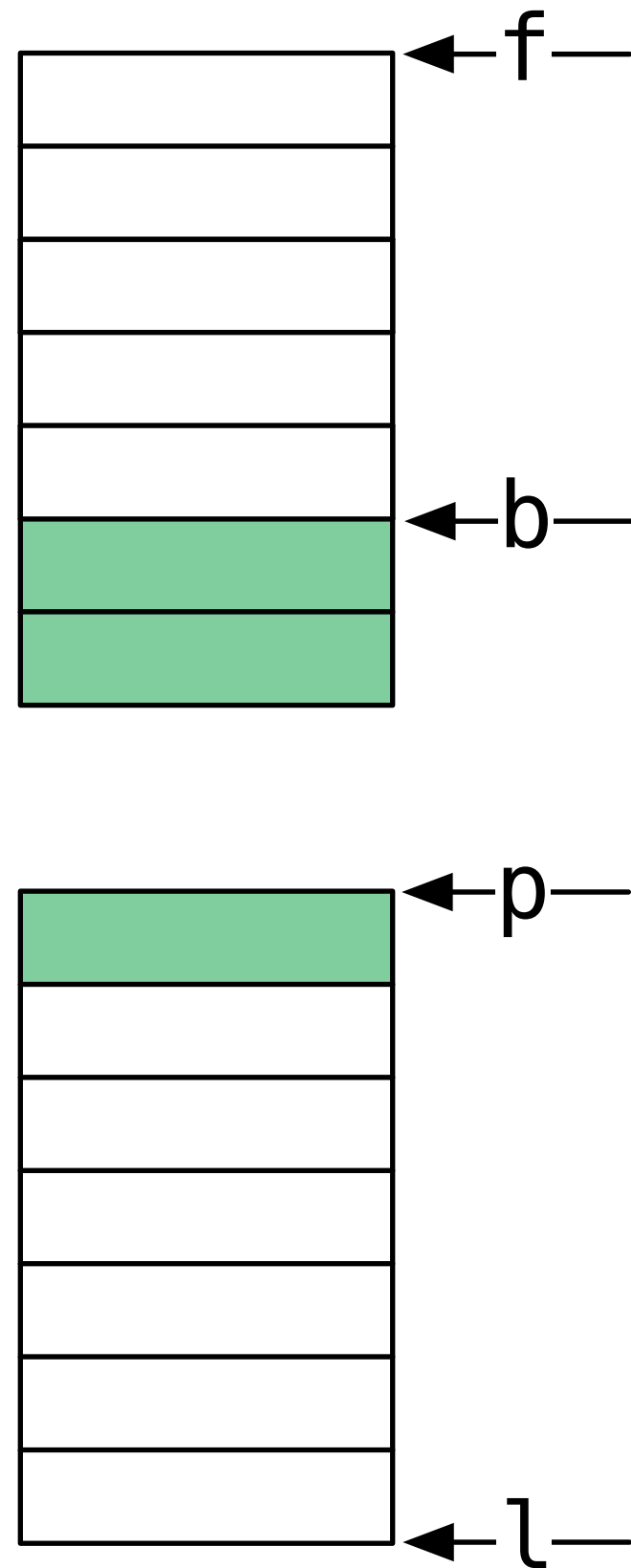
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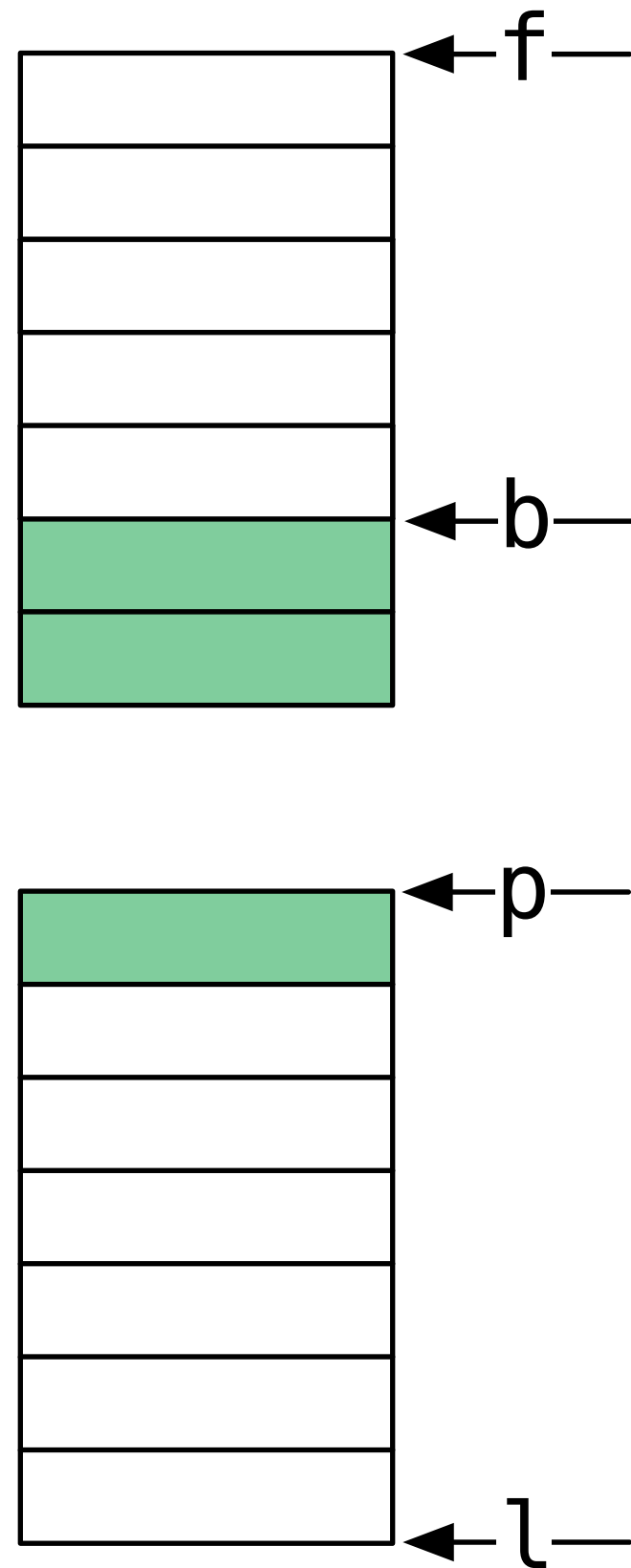
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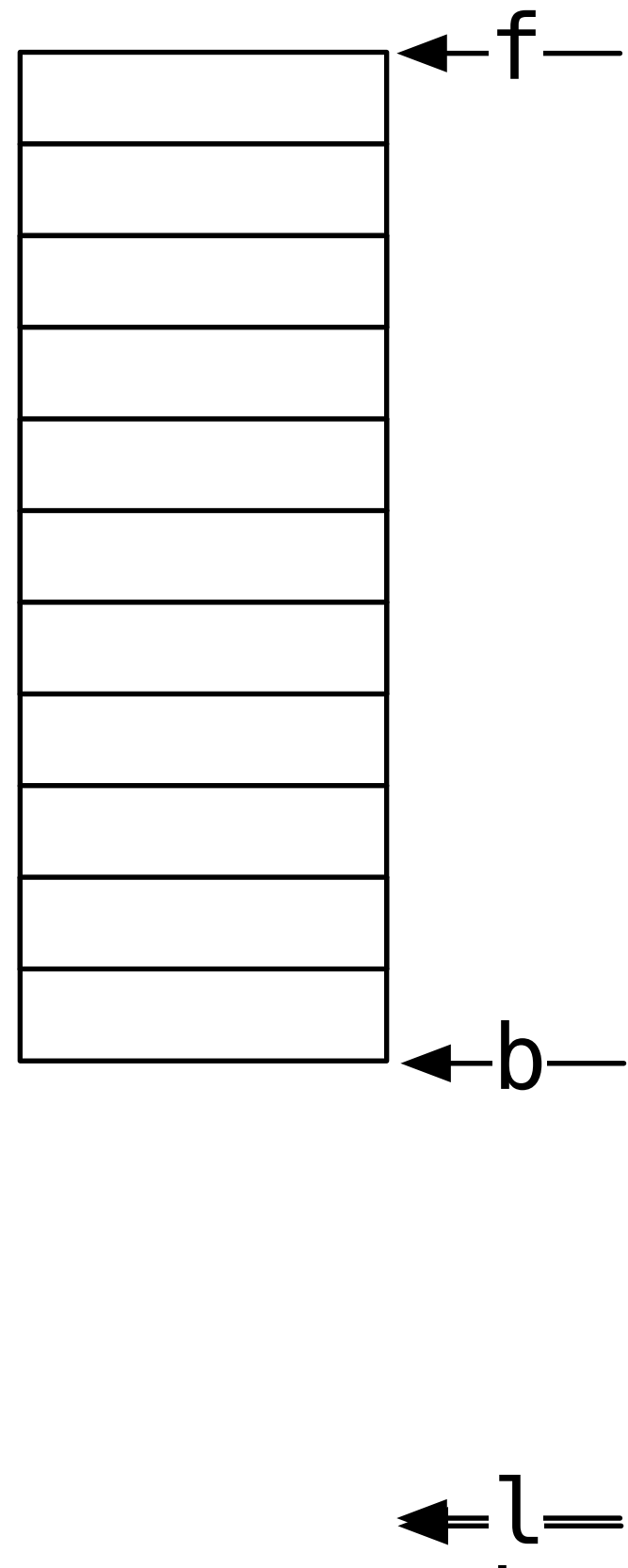
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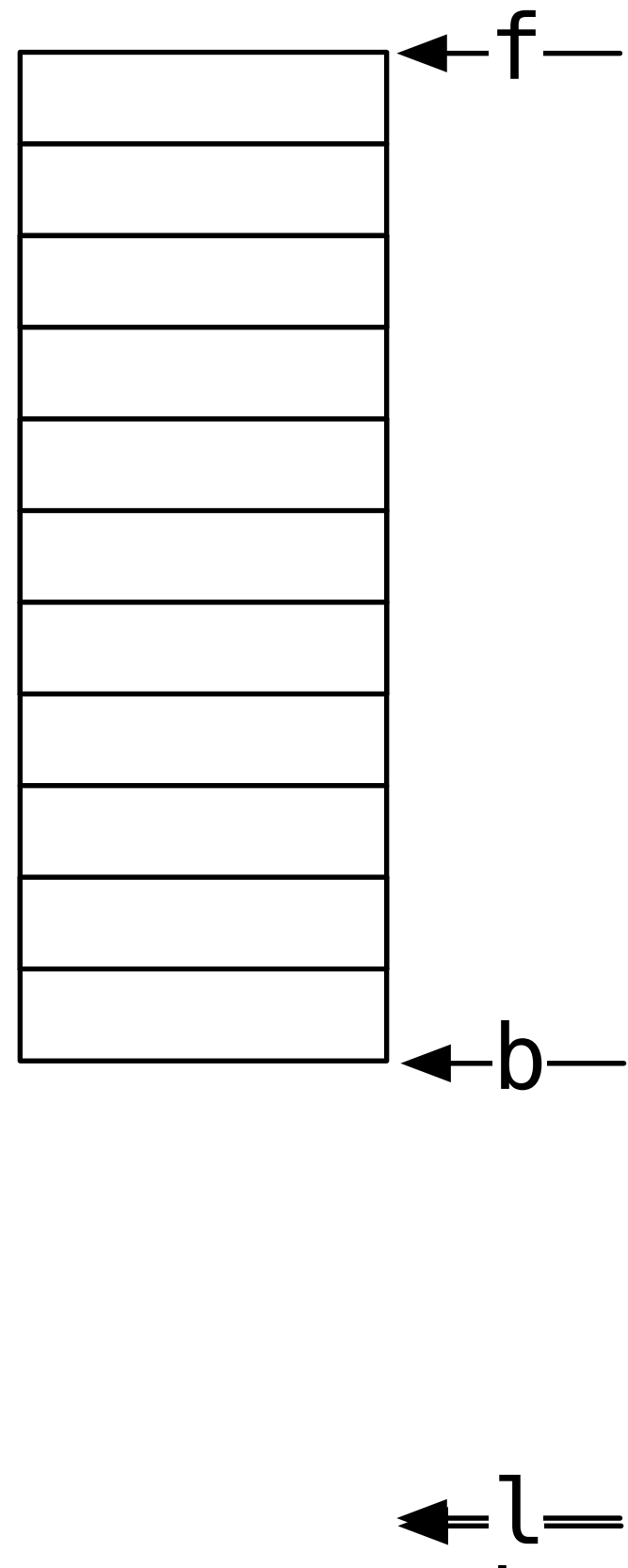
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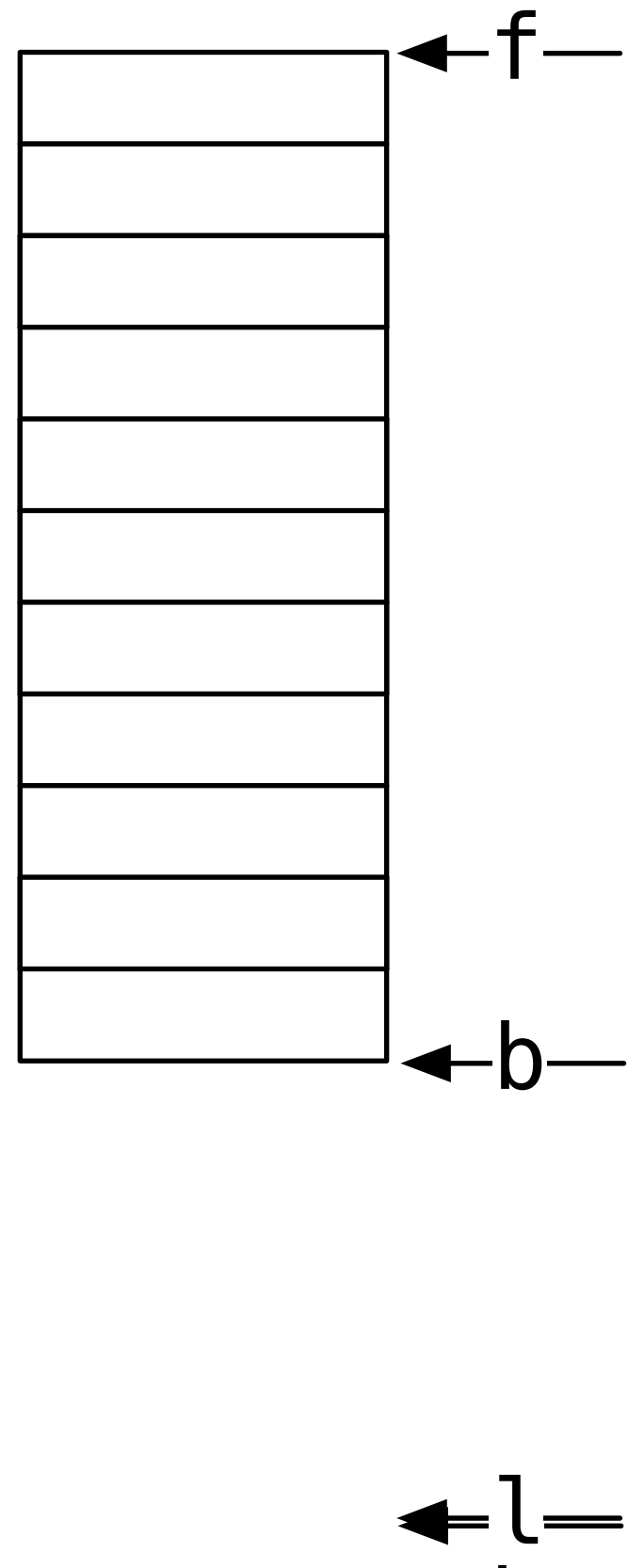
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    while (p != l) {
        if (*p != a) {
            *b = std::move(*p);
            ++b;
        }
        ++p;
    }
    return b;
}
```

Remove

```
/**  
    Removes values equal to `a` in the range `[f, l)`.  
  
    \return the position, `b`, such that `[f, b)` contains all the  
            values in `[f, l)` not equal to `a`  
  
            values in `[b, l)` are unspecified  
*/
```

```
template <std::forward_iterator I, class T>  
auto remove(I f, I l, const T& a) -> I;
```

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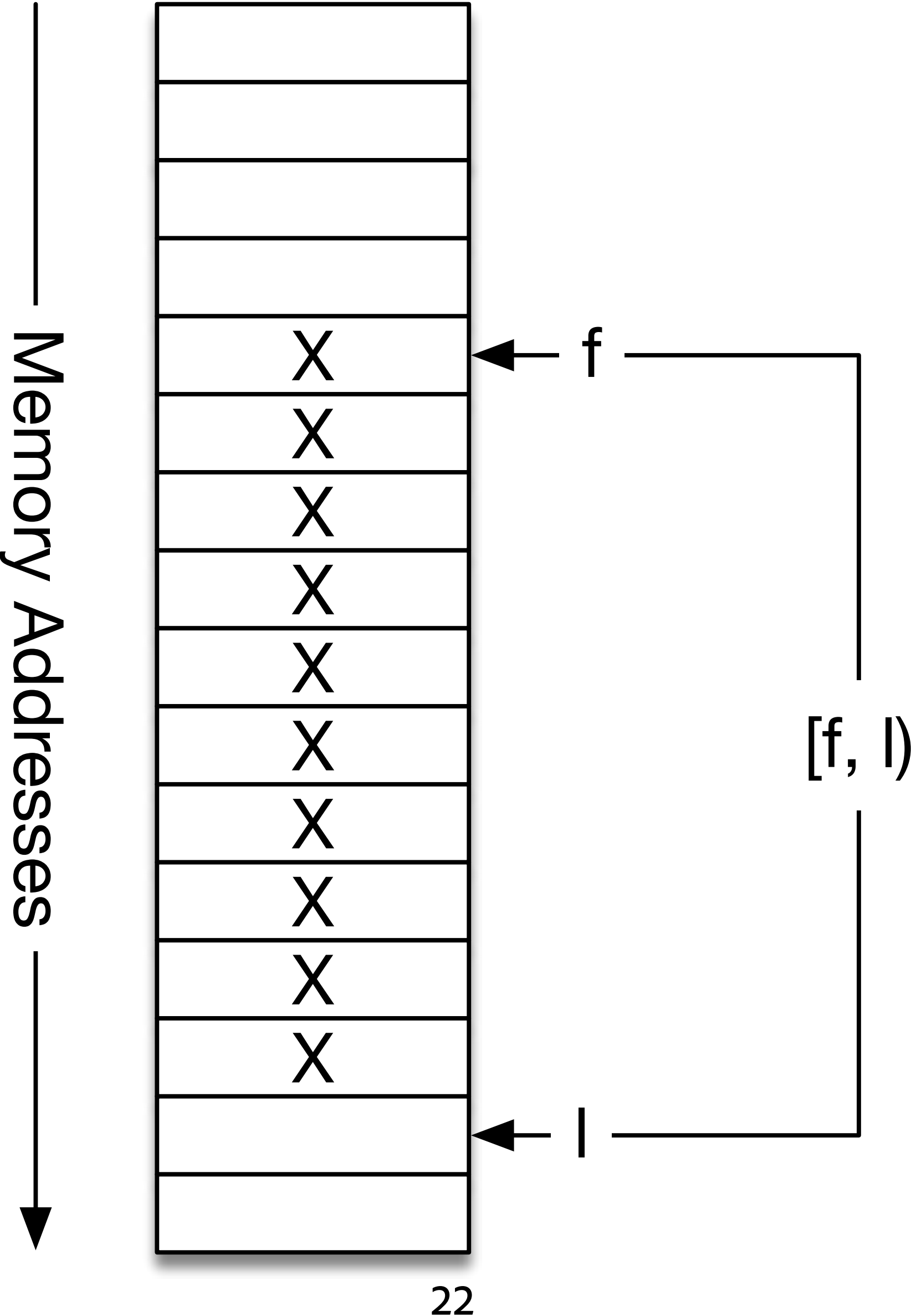
Sequences

- For a sequence of n elements, there are $n + 1$ positions
- Ways to represent a range of elements
 - Closed interval $[f, l]$
 - Open interval (f, l)
 - Half-open interval $[f, l)$
 - By strong convention, open on the right

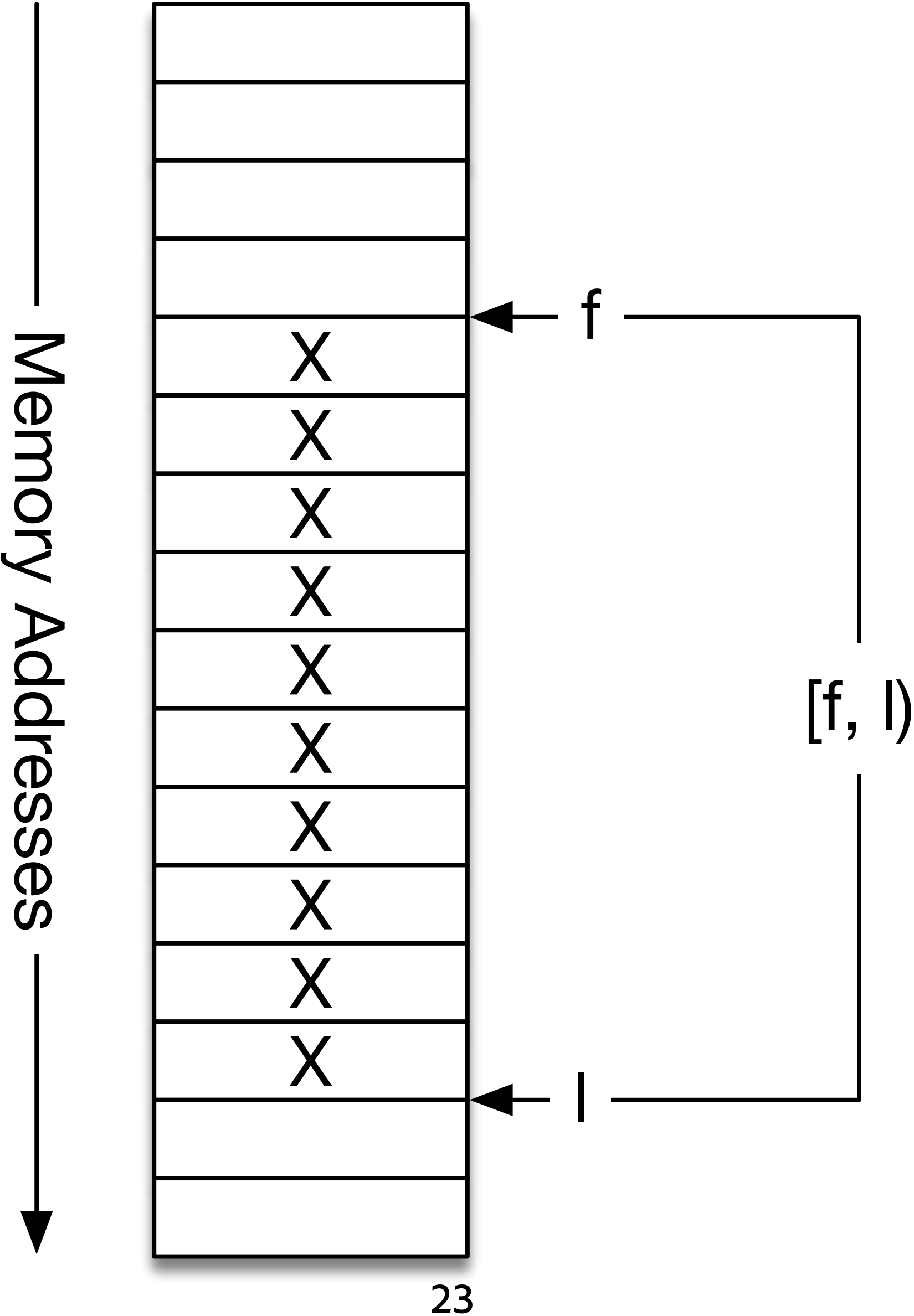
Half-Open Intervals

- $[p, p)$ represents an empty range at position p
 - All empty ranges are not equal
- Cannot express the last item in a set with positions of the same set type
 - i.e., `[INT_MIN, INT_MAX]` is not expressible as a half-open interval with type `int`
- Think of the positions as the lines between the elements

Half-Open Intervals



Half-Open Intervals



Half-Open Intervals

- In this model, there is a symmetry with reverse ranges $(l, f]$
 - The dereference operation is asymmetric. dereferencing at a position p is the value in $[p, p + 1)$
- Half-open intervals avoid off-by-one errors and confusion about *before* or *after*
- In C and C++, half-open intervals are built into the language. For any object, a , $\&a$ is a pointer to the object, and $\&a + 1$ is a valid pointer but may not be dereferenceable.
- Any object can be treated as a range of one element

```
int a{42};  
copy(&a, &a + 1, ostream_iterator<int>(cout));  
42
```

Half-Open Intervals

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 - unbounded: $[f, \dots)$, limit is dependent on an extrinsic relationship
 - i.e., the range is require to be the same length or greater than another range

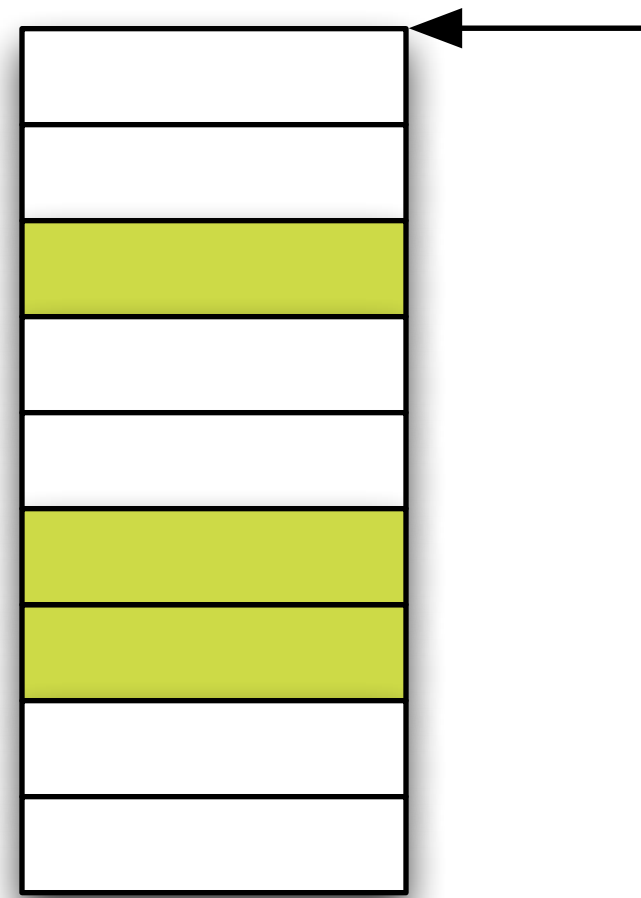
Gather



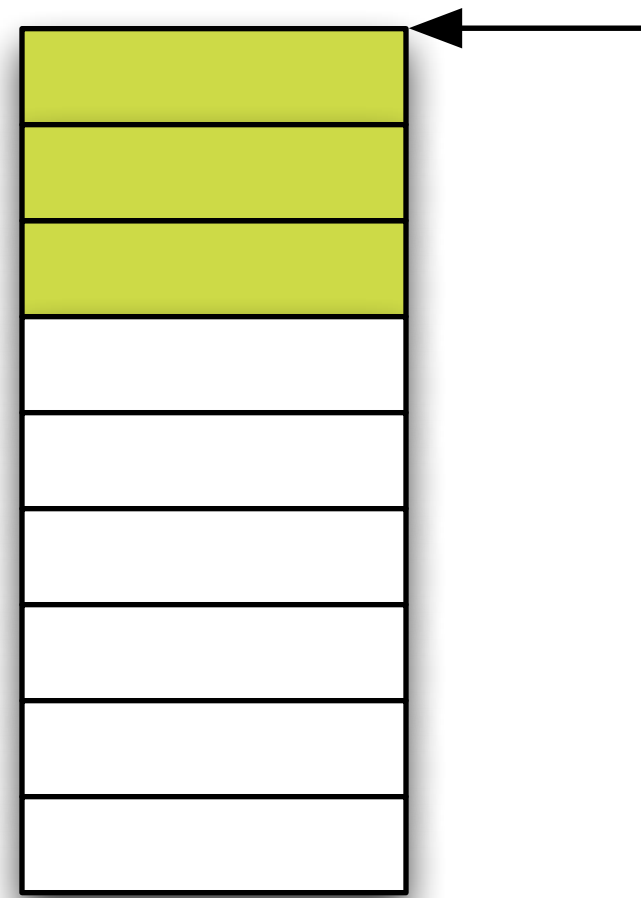
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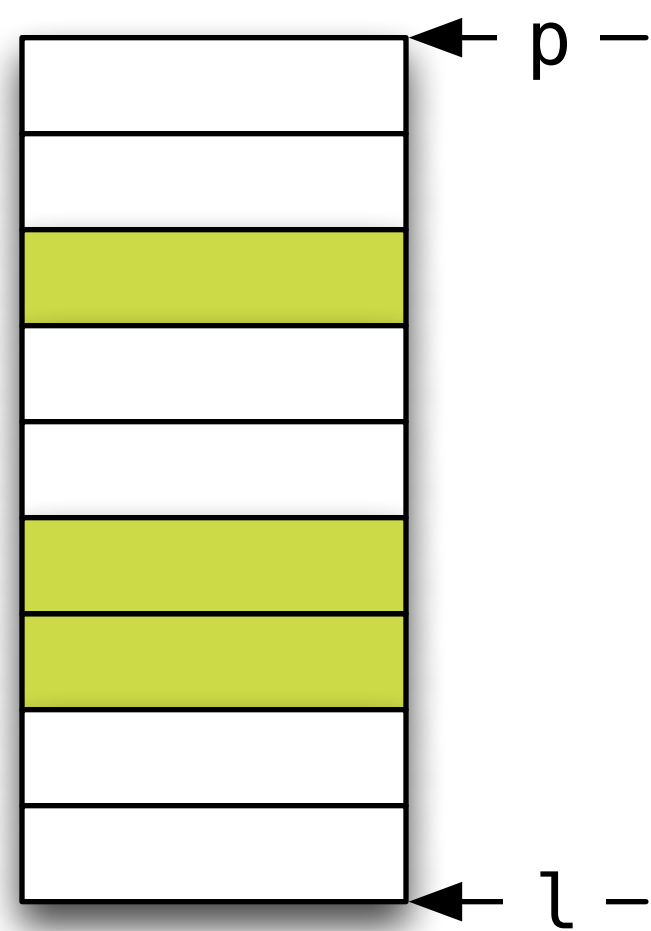
Composing Algorithms - Gather



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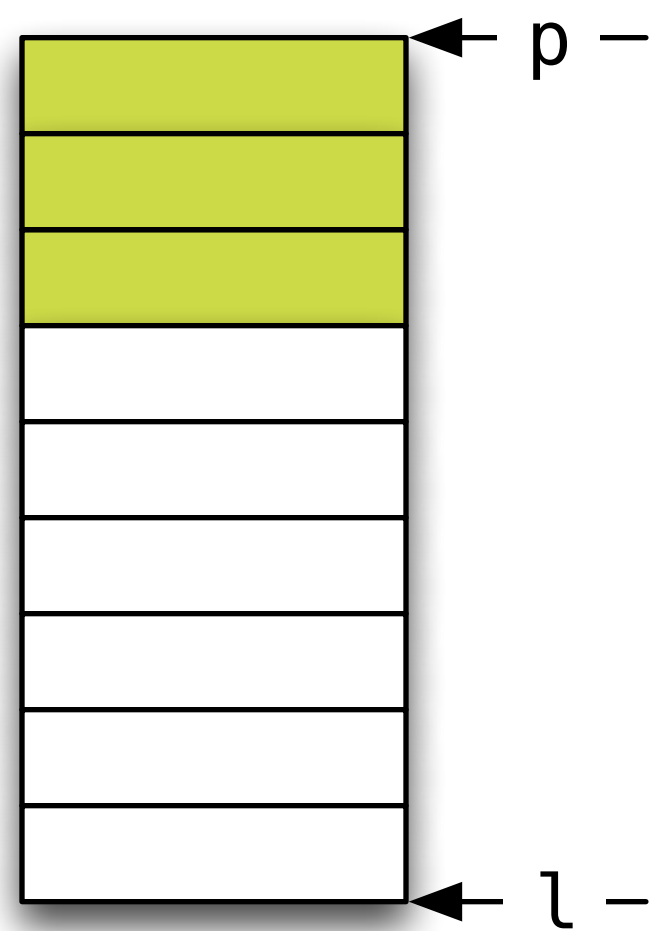


Composing Algorithms - Gather



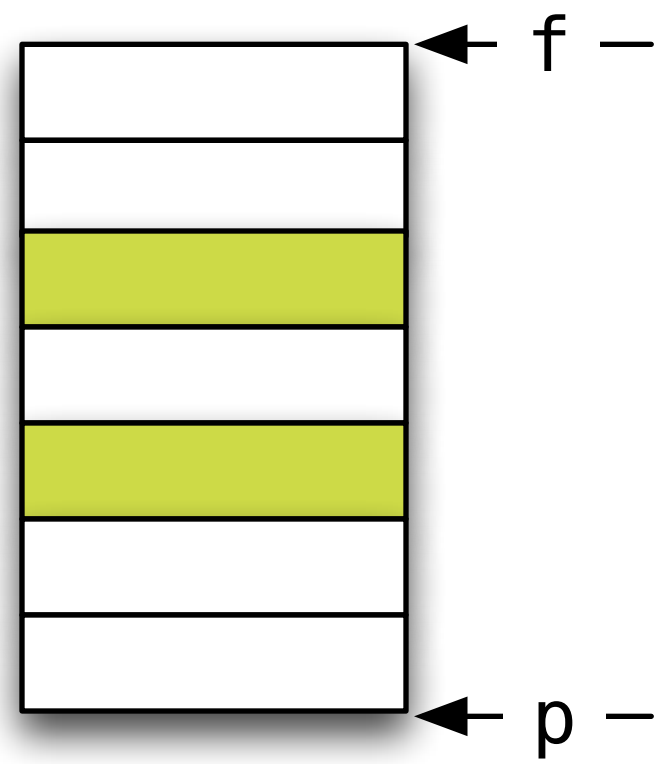
`stable_partition(p, l, s)`

Composing Algorithms - Gather



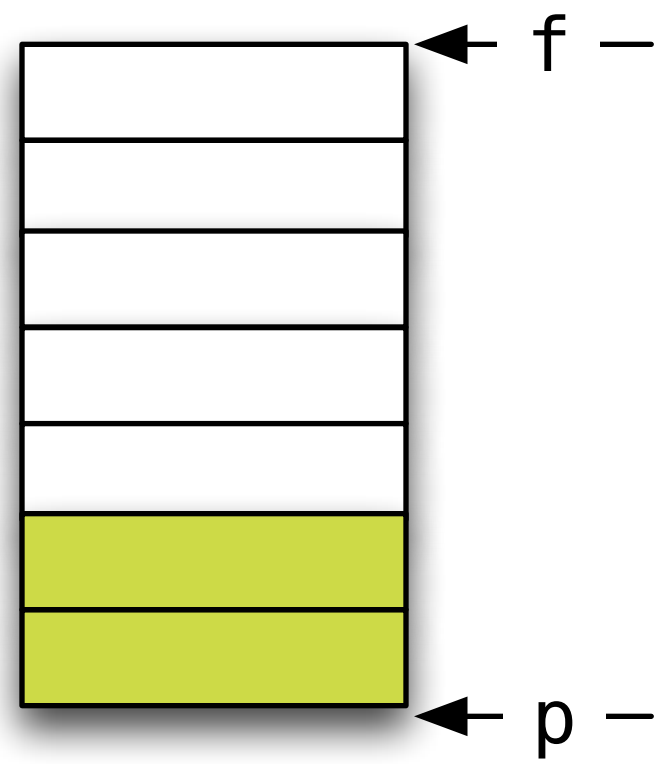
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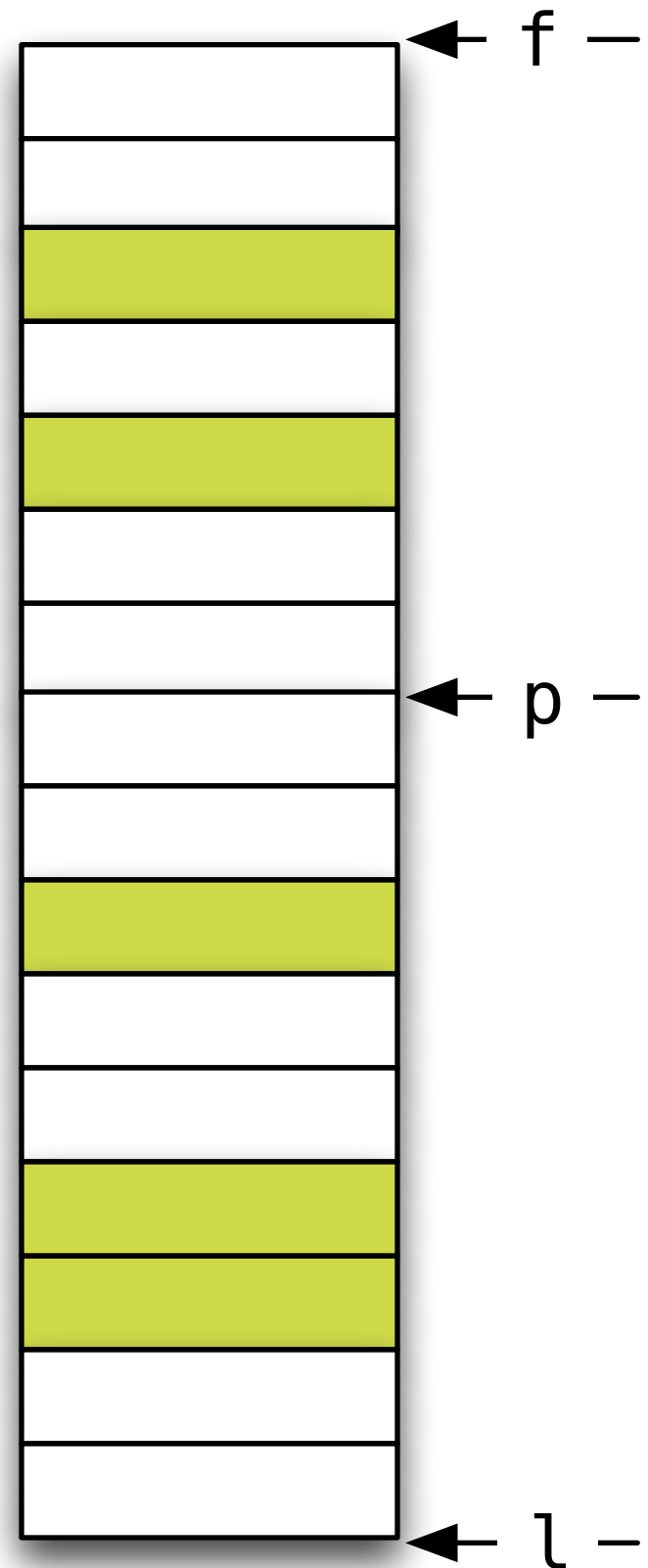
```
stable_partition(f, p, not1(s))
```

Composing Algorithms - Gather



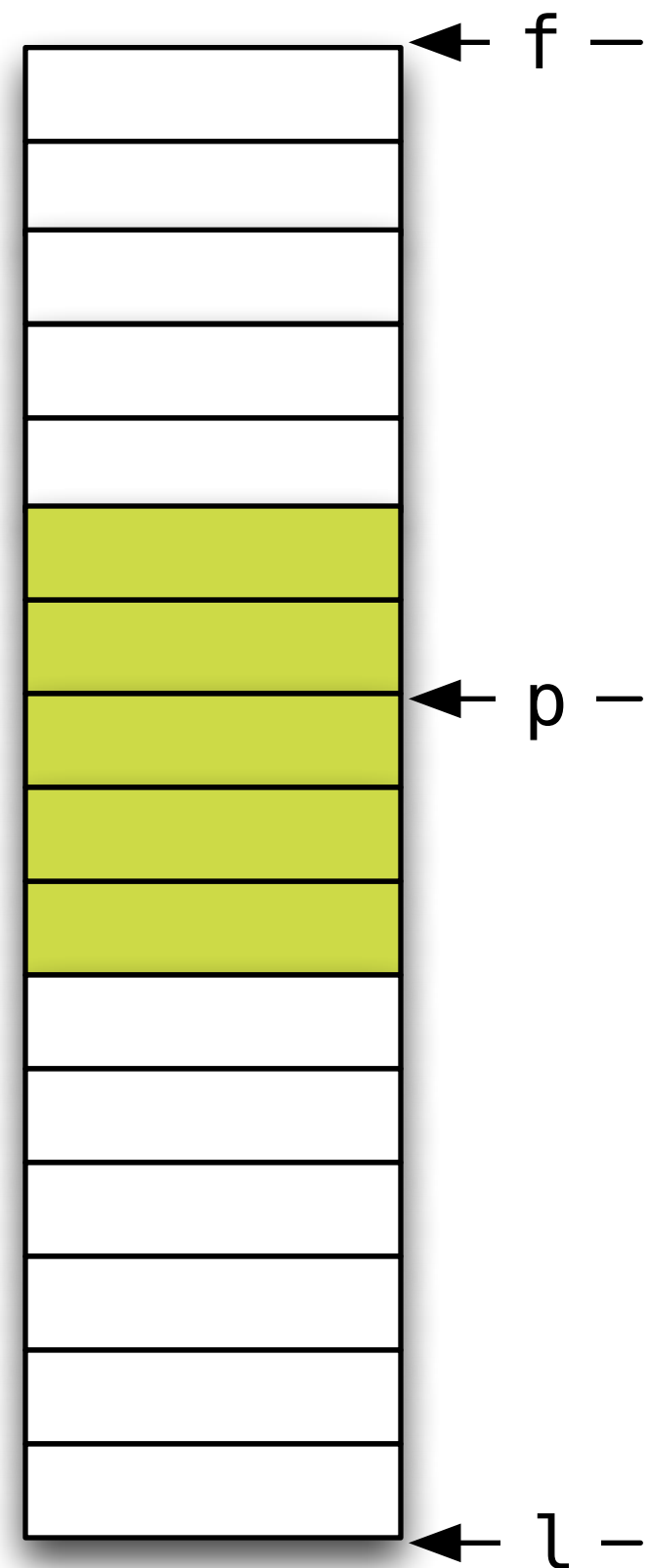
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Composing Algorithms - Gather



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stable_partition(f, p, not1(s))  
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Composing Algorithms - Gather



```
return { stable_partition(f, p, not1(s)),  
        stable_partition(p, l, s) };
```

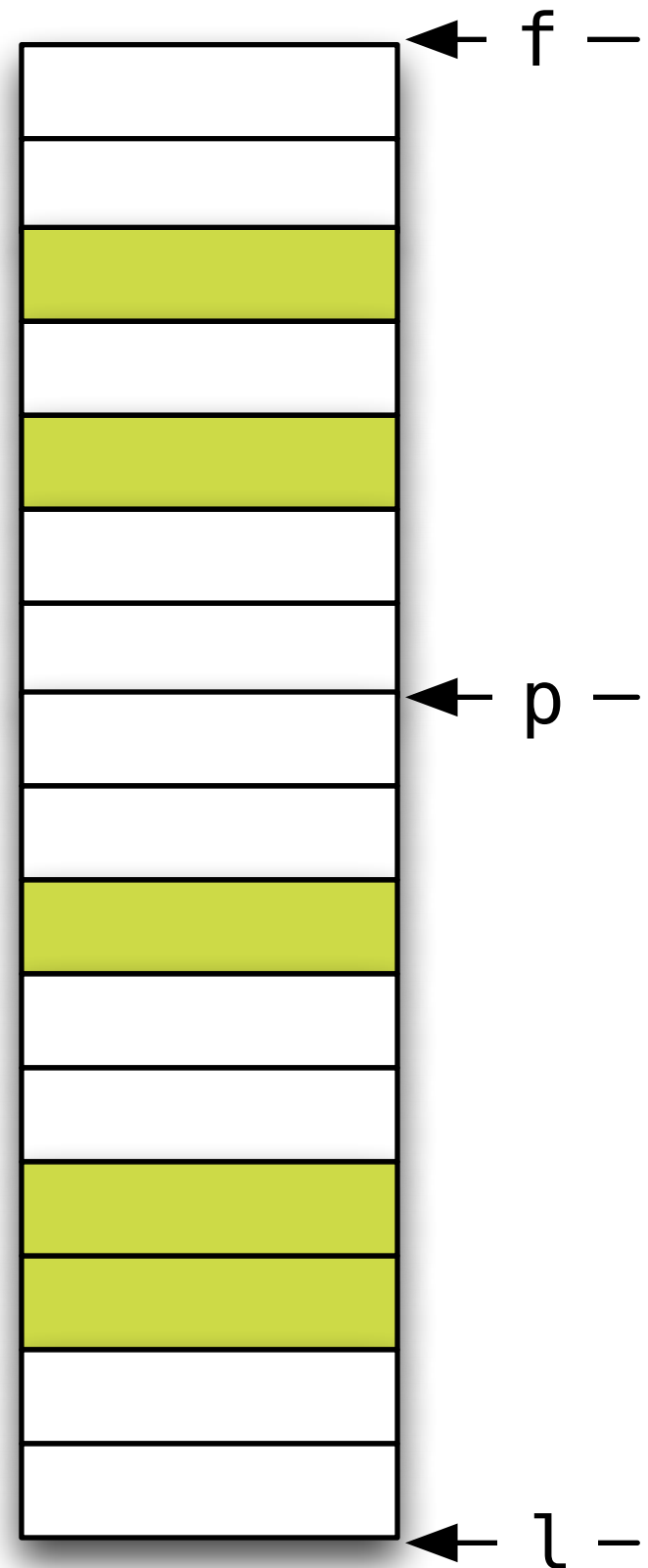

Composing Algorithms - Gather



```
/// Gather elements in [f, l) satisfying s at p
/// and returns range containing those elements
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```
template <class I, // BidirectionalIterator
          class S> // UnaryPredicate
auto gather(I f, I l, I p, S s) -> pair<I, I>
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    return { stable_partition(f, p, not1(s)),
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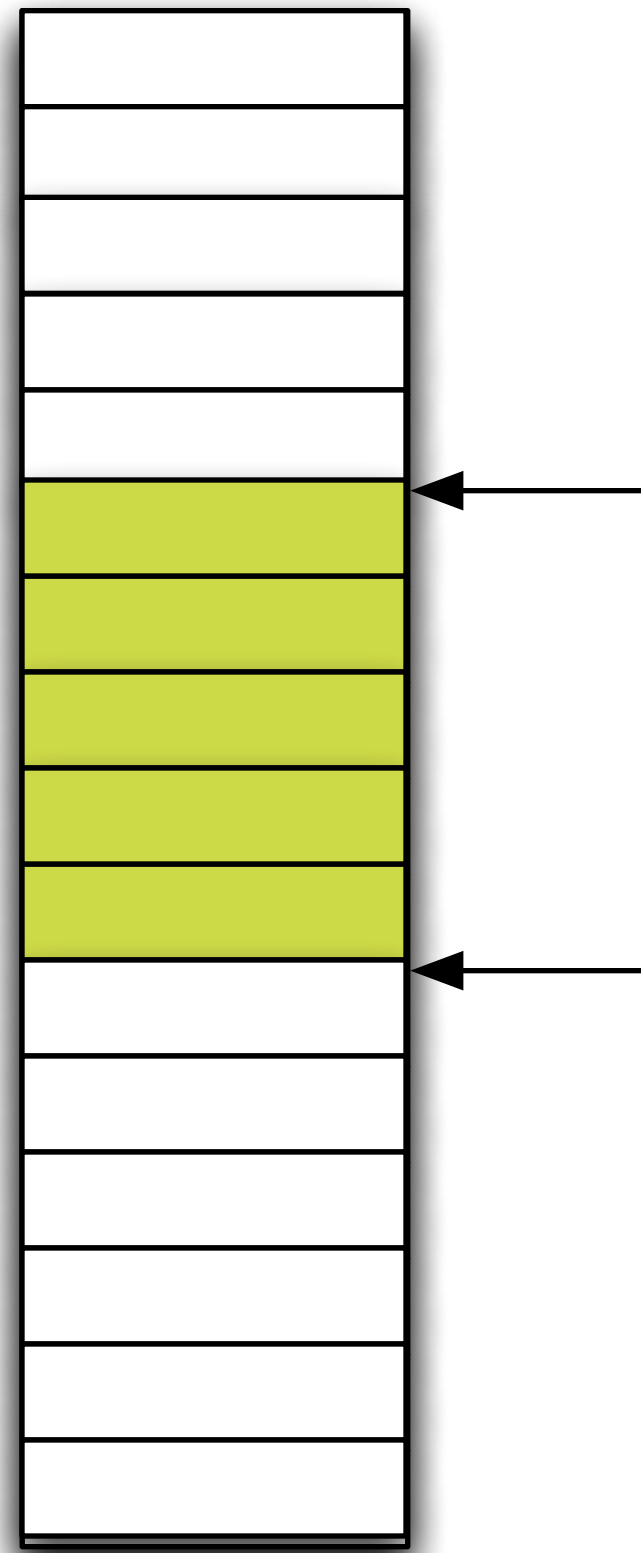
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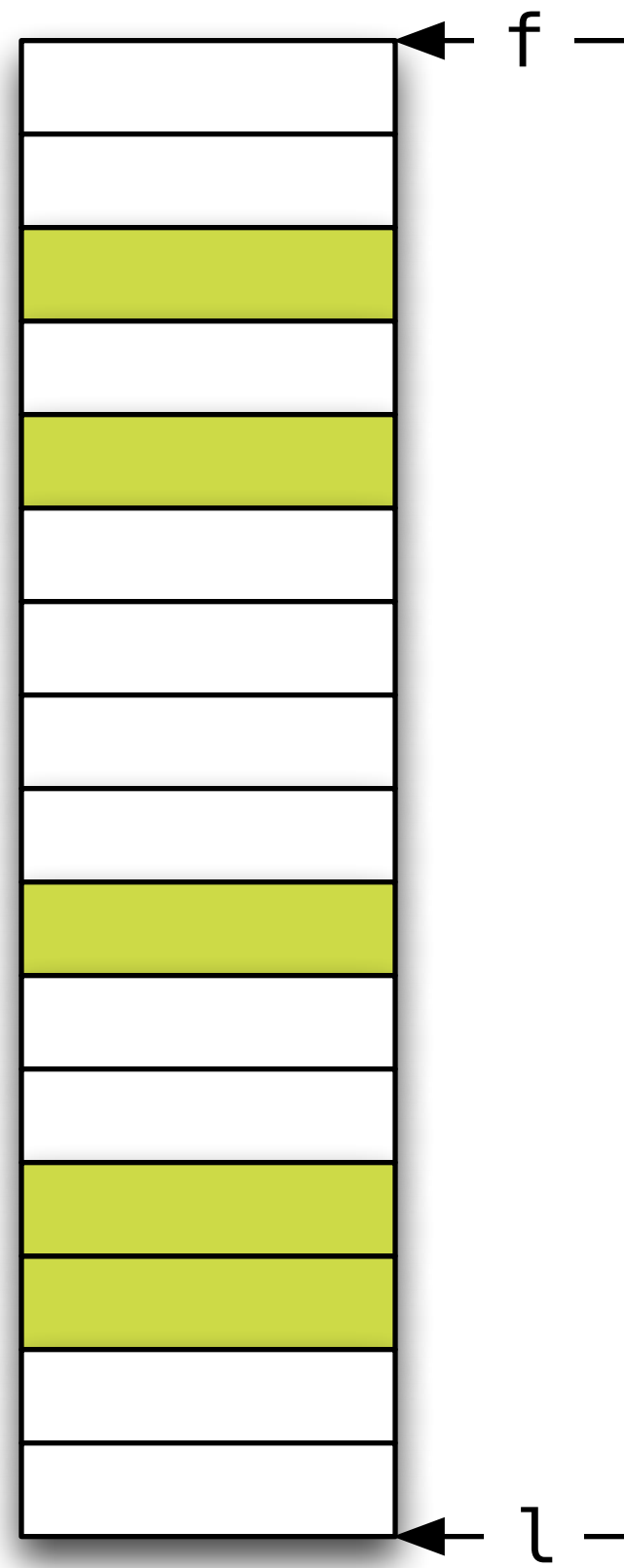
Composing Algorithms - Gather



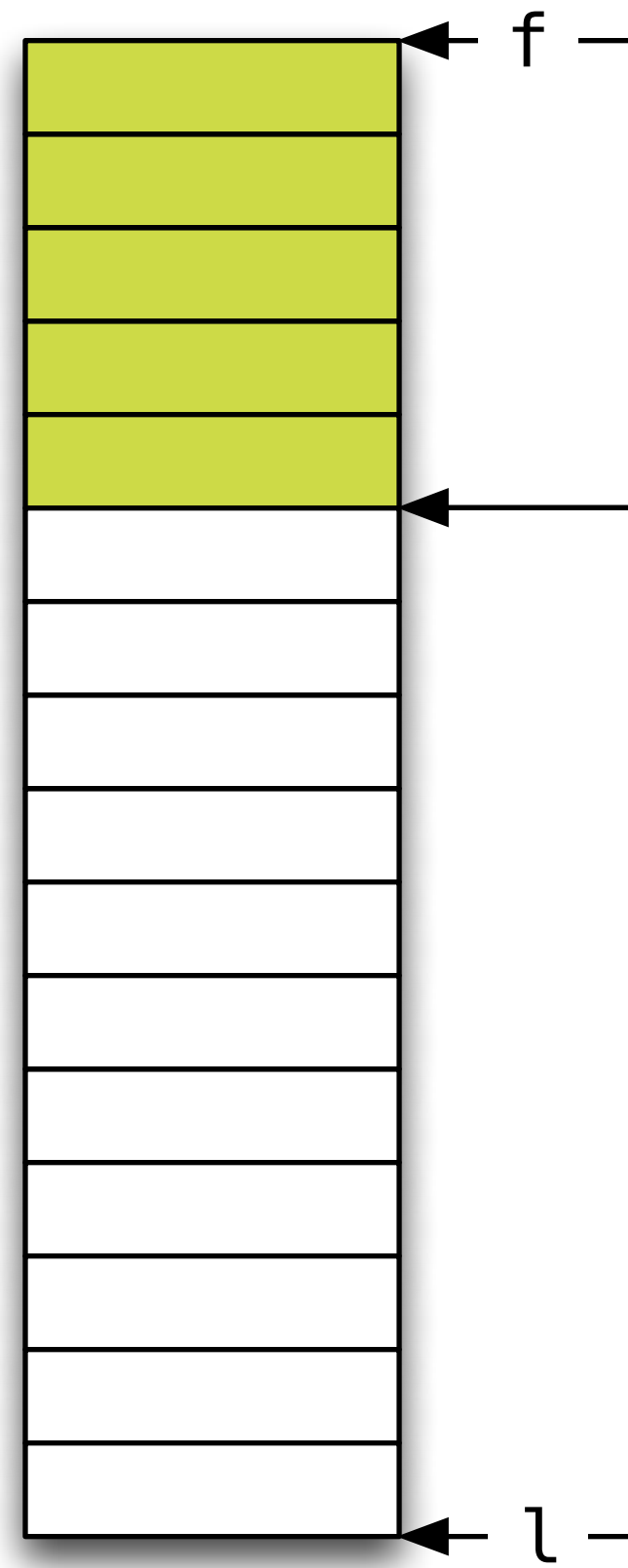
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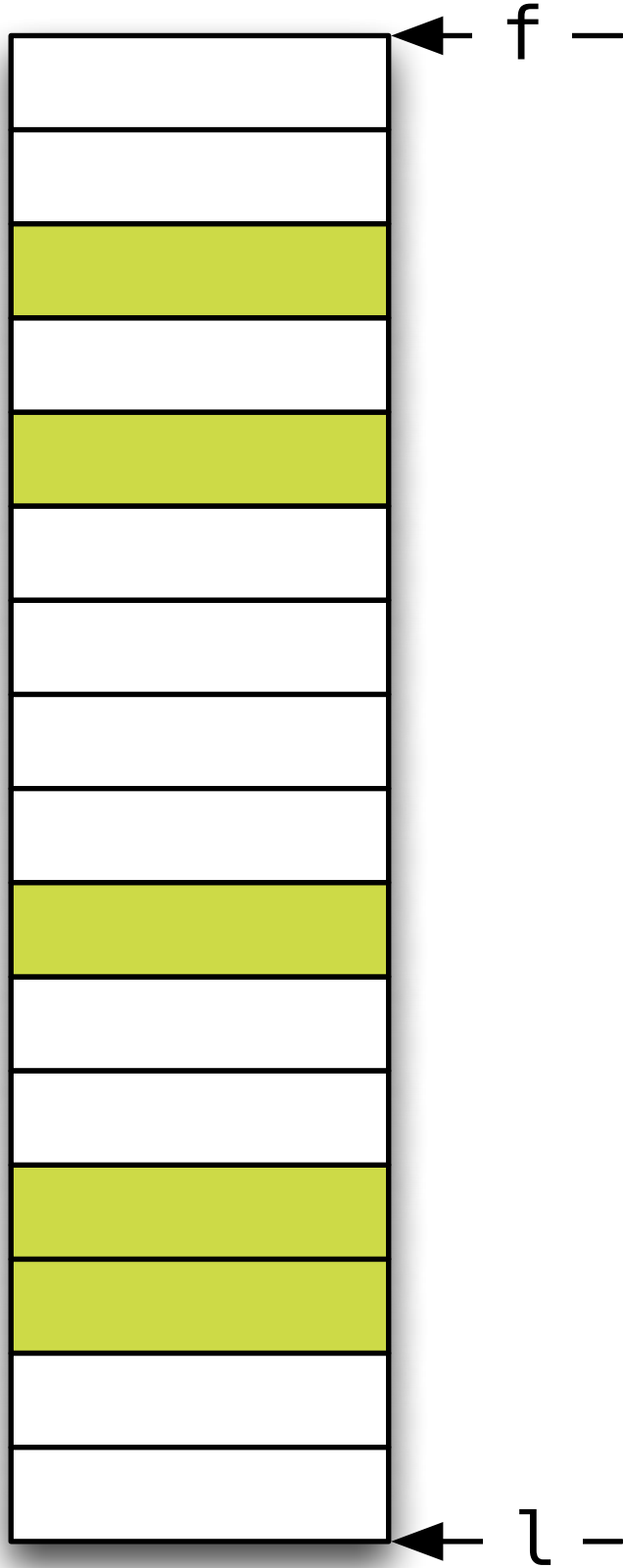
Composing Algorithms - Stable Partition



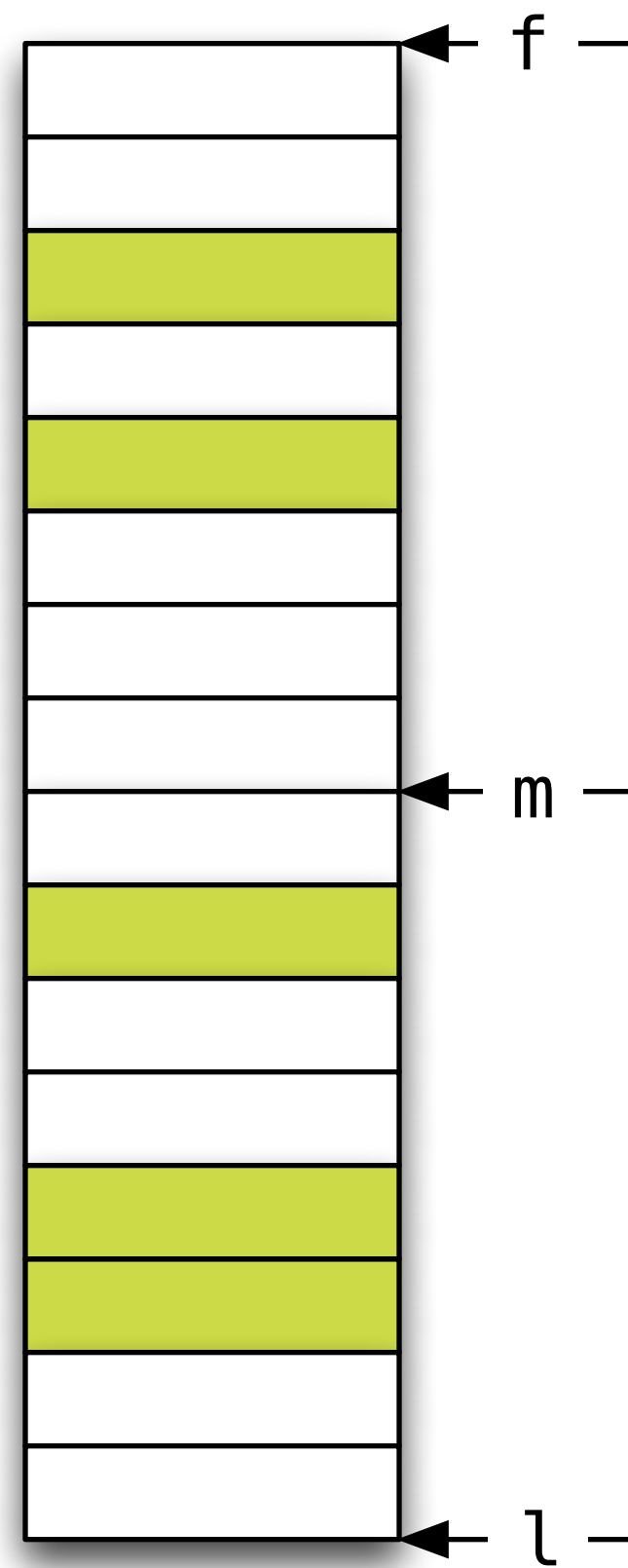
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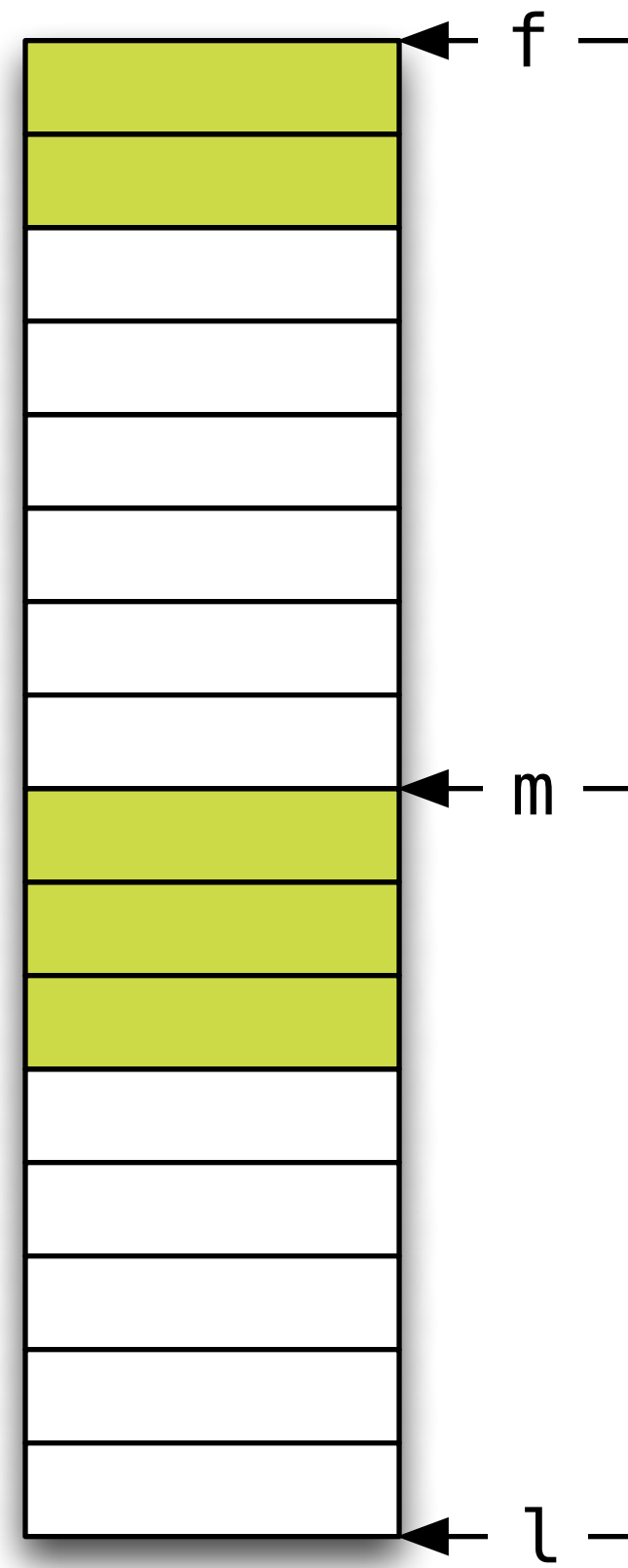
Composing Algorithms - Stable Partition



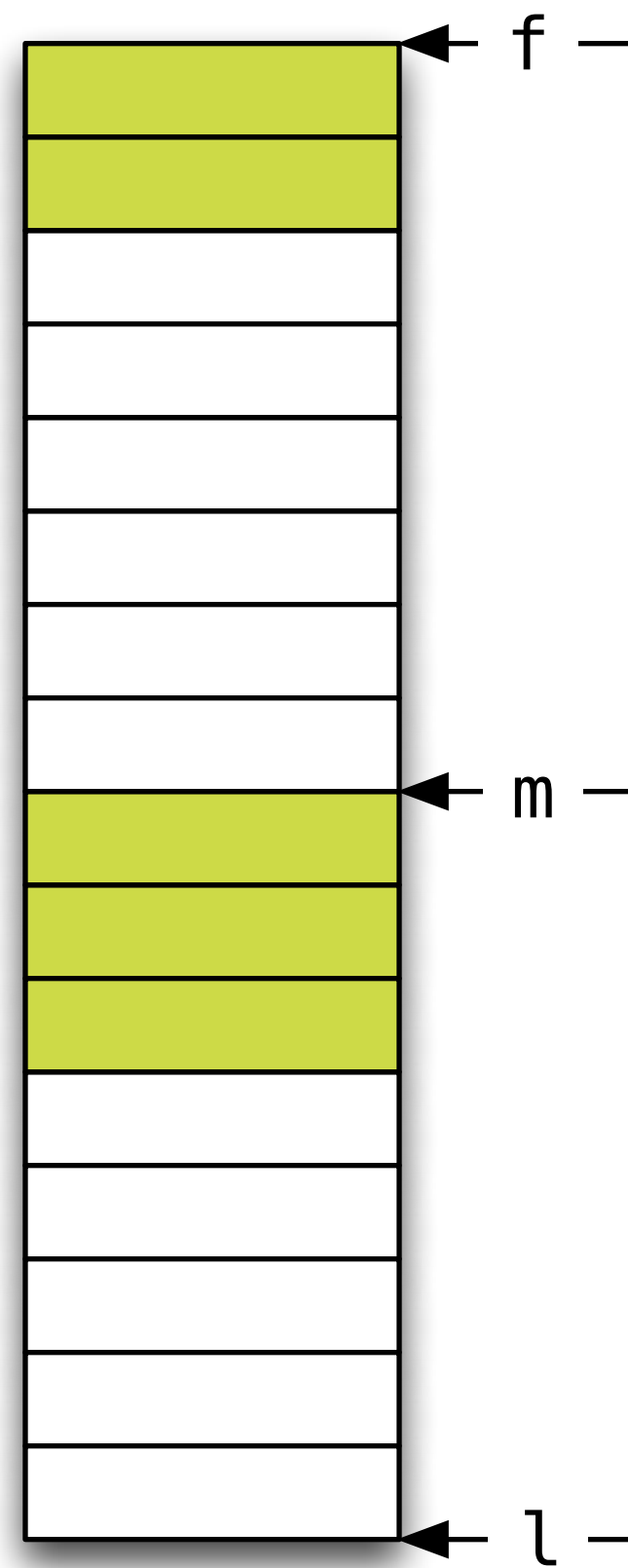
Composing Algorithms - Stable Partition



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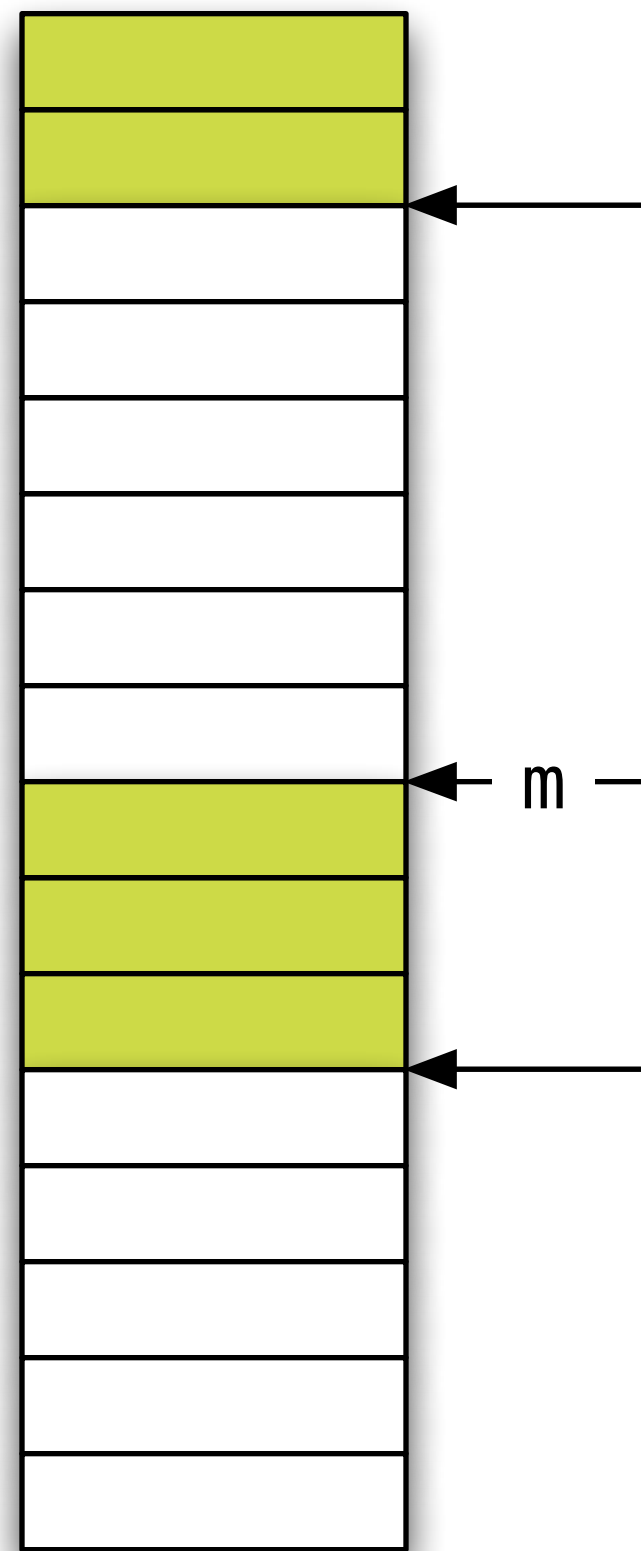
Composing Algorithms - Stable Partition



```
stable_partition(f, m, p)
```

```
stable_partition(m, l, p)
```

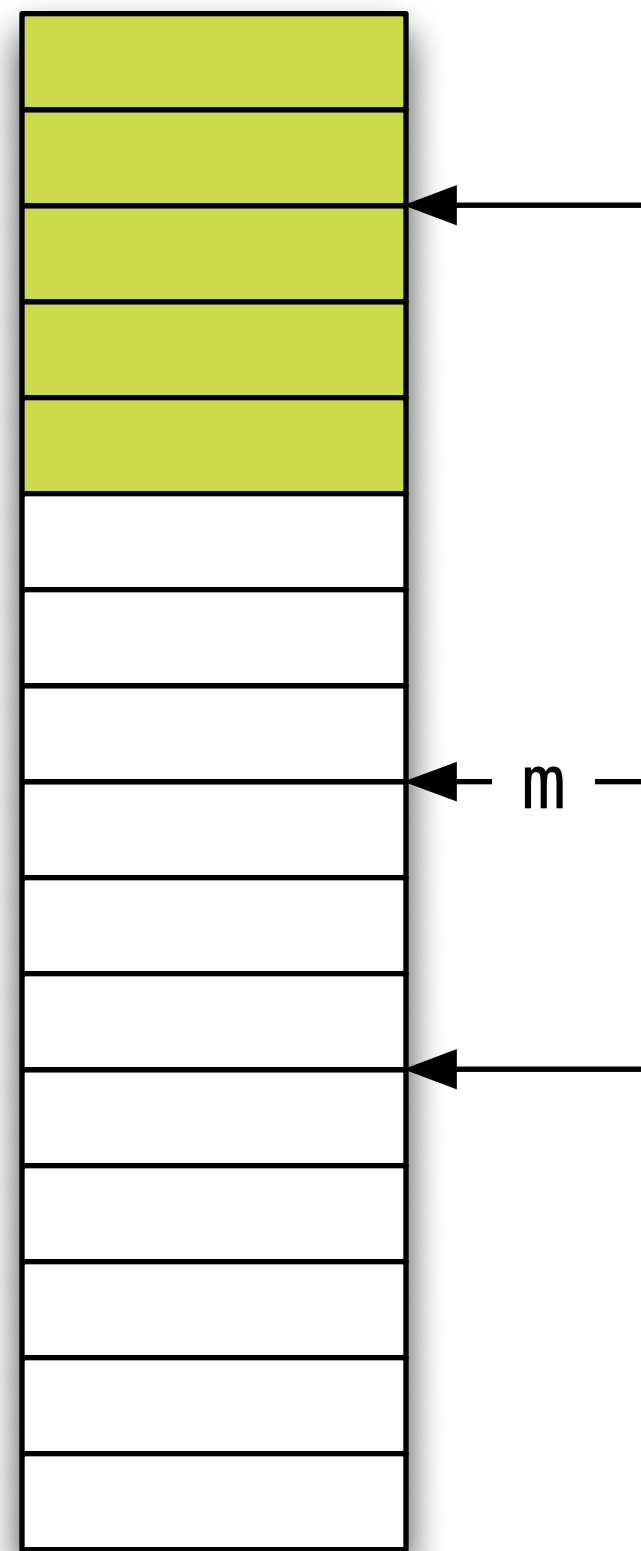
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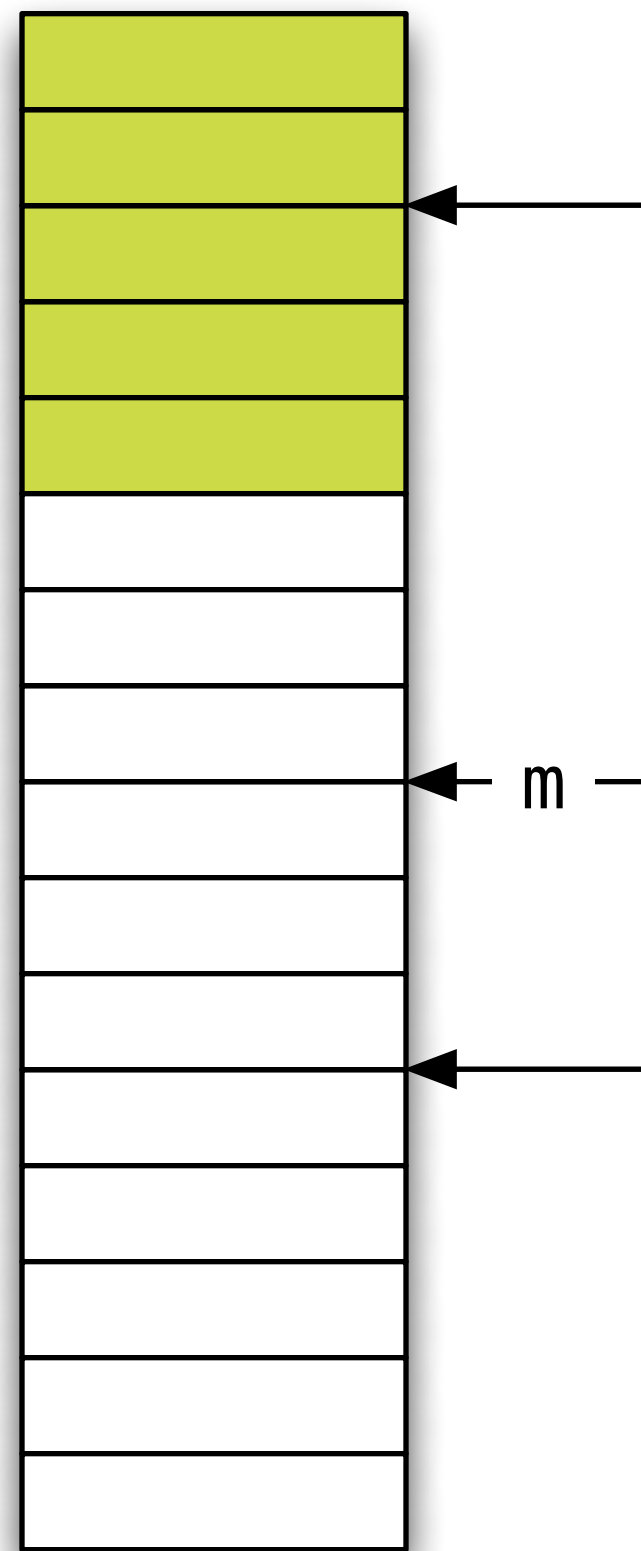
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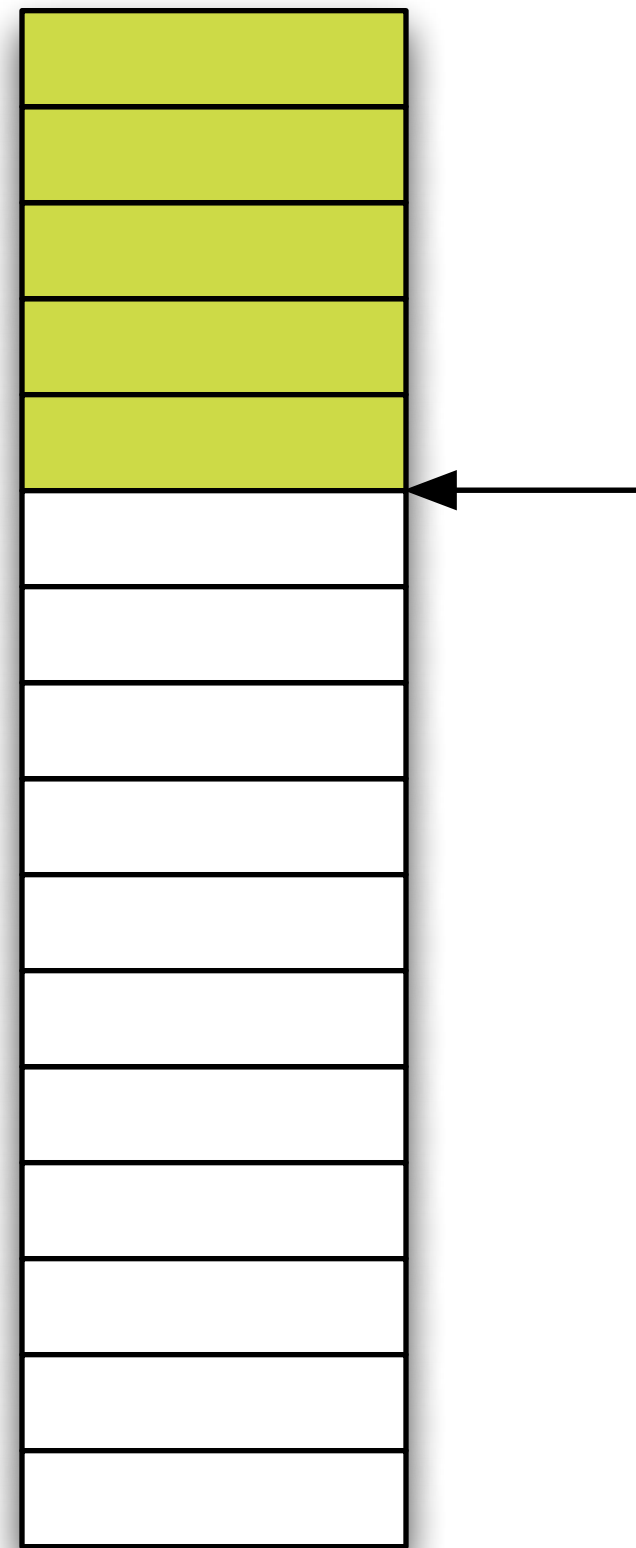
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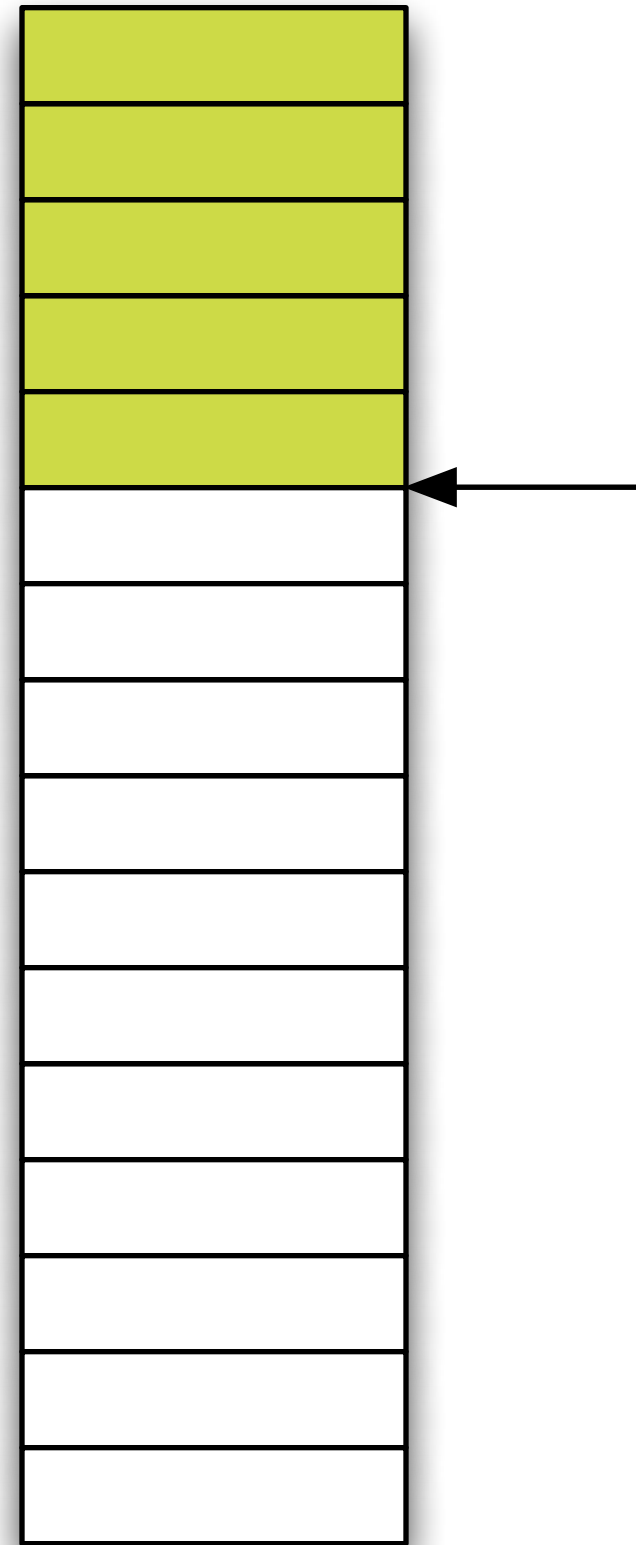
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rotate(stable_partition(f, m, p),  
       m,  
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```

Composing Algorithms - Stable Partition



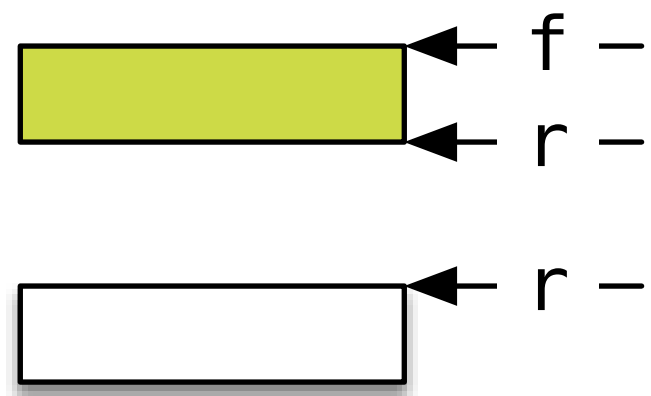
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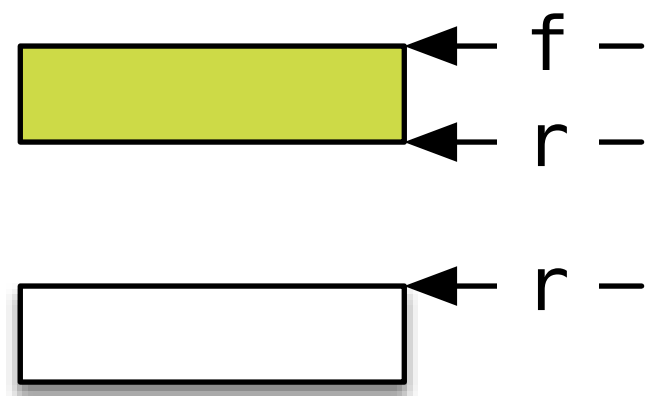
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return rotate(stable_partition(f, m, p),  
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Composing Algorithms - Stable Partition



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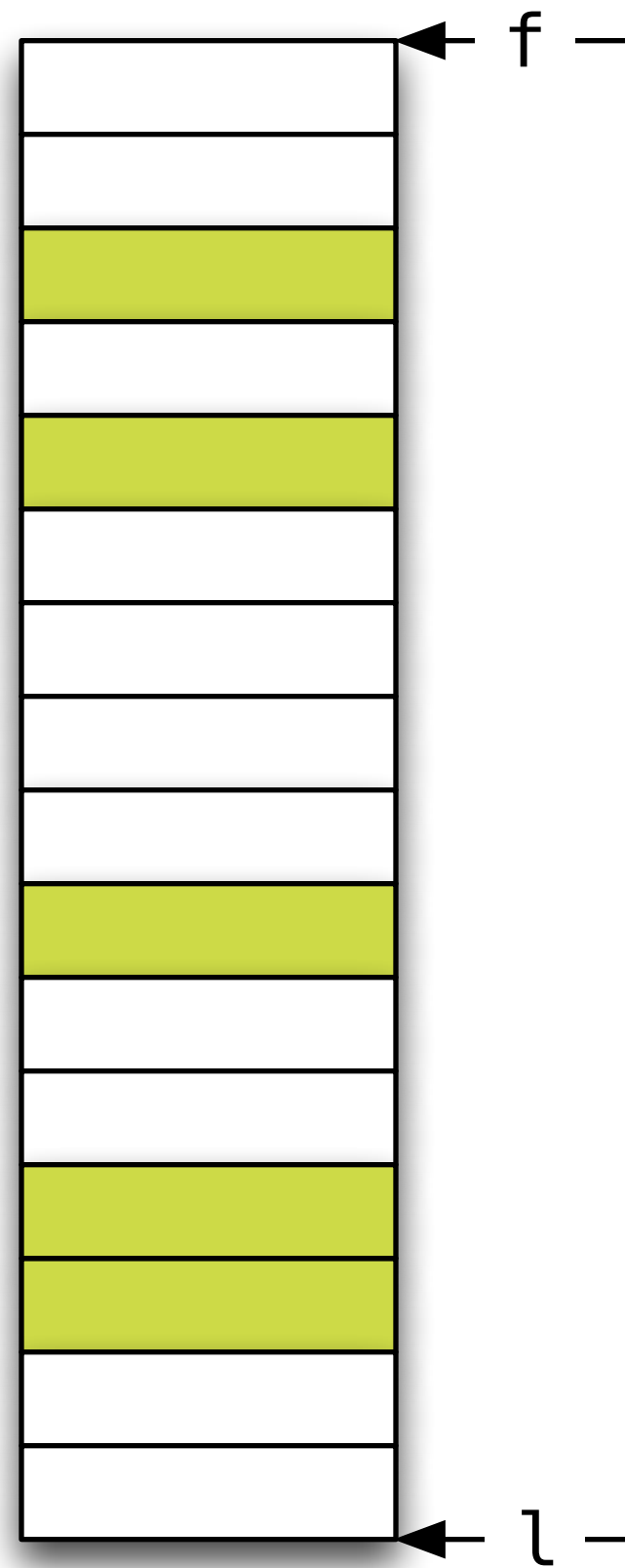

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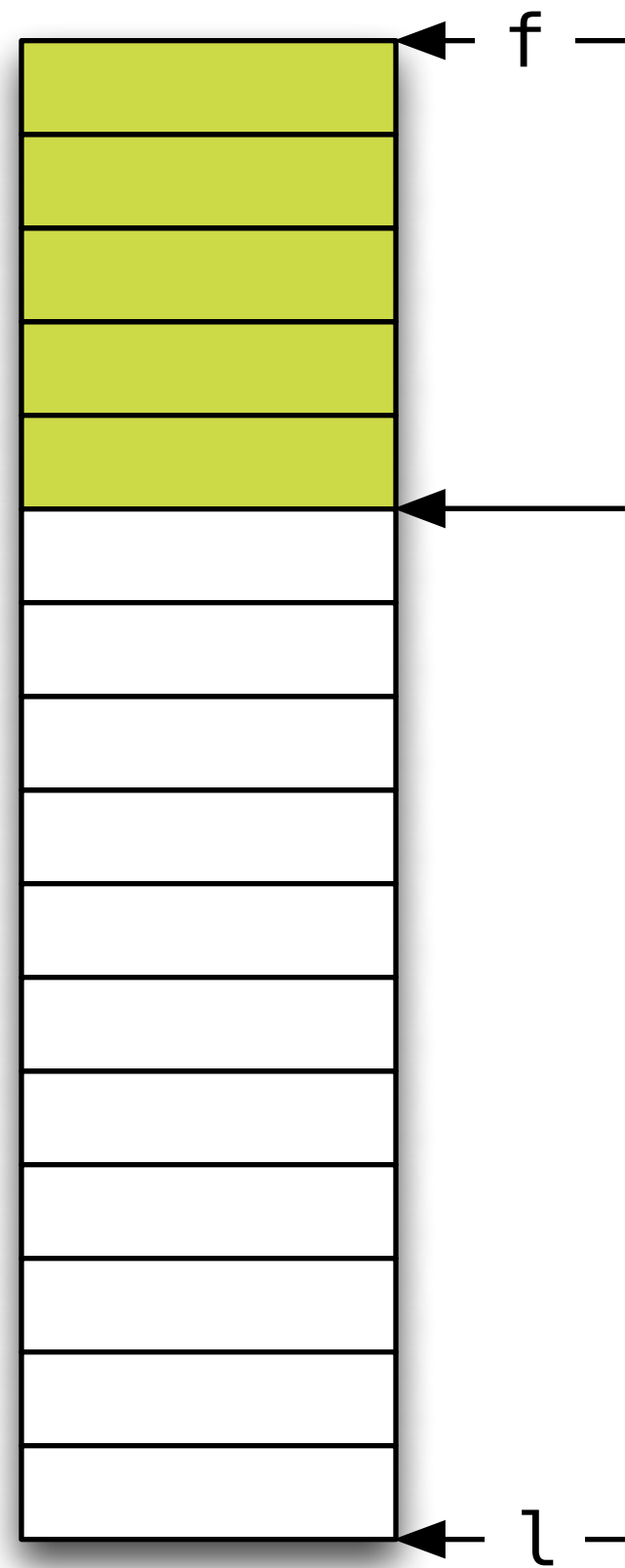


```
template <class I, // ForwardIterator
          class P> // UnaryPredicate
auto stable_partition(I f, I l, P p) -> I
{
    auto n = l - f;
    if (n == 0) return f;
    if (n == 1) return f + p(*f);

    auto m = f + (n / 2);

    return rotate(stable_partition(f, m, p),
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                  stable_partition(m, l, p));
}
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Composing Algorithms - Stable Partition



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 - i.e., $a < b$ implies $position(a) < position(b)$

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∅ Patents

Q & A



About the artist

Dan Zucco

London-based 3D art and motion director Dan Zucco creates repeating 2D patterns and brings them to life as 3D animated loops. Inspired by architecture, music, modern art, and generative design, he often starts in Adobe Illustrator and builds his animations using Adobe After Effects and Cinema 4D. Zucco's objective for this piece was to create a geometric design that felt like it could have an infinite number of arrangements.

Made with

 Adobe Illustrator

 Adobe After Effects



Adobe

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Artwork by Dan Zucco