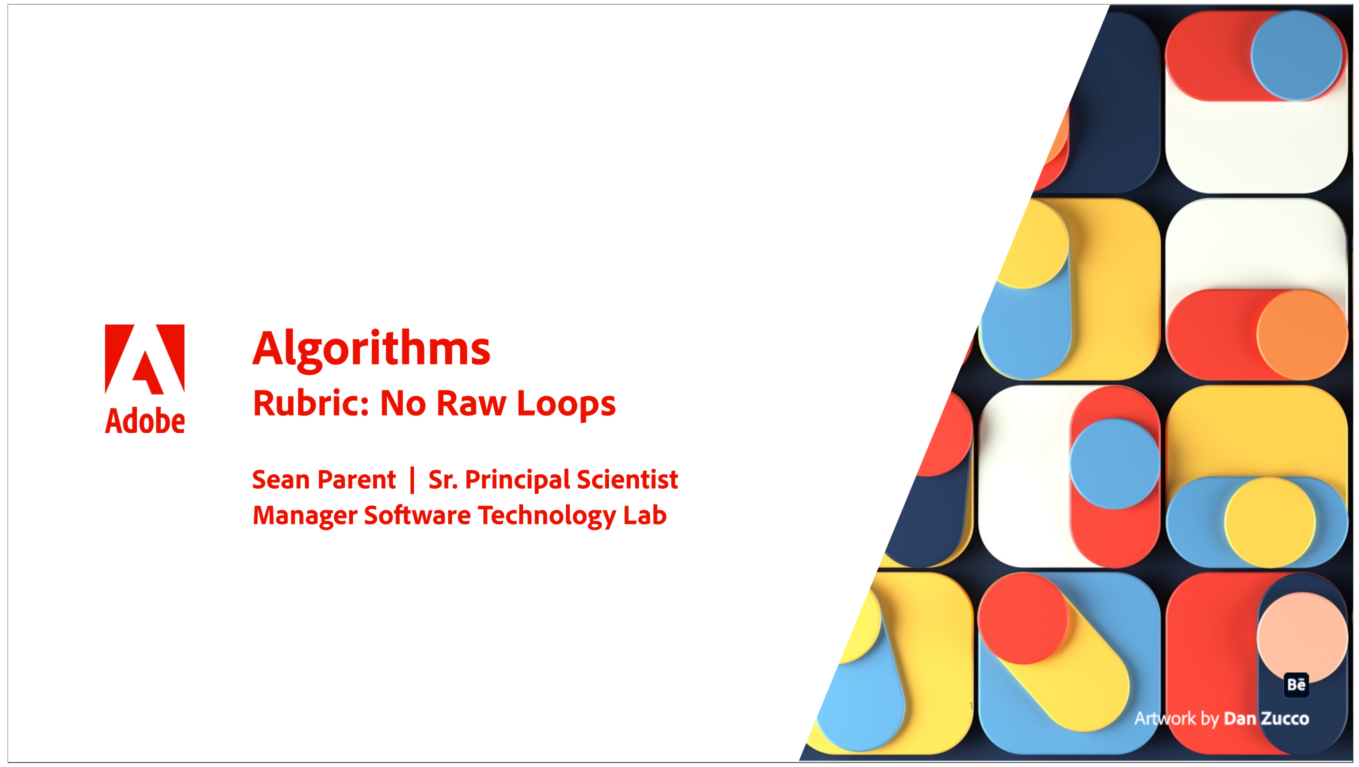

This talk will be a little tight - and I don't want to spill into a "part 2," so I ask you to hold questions until the end. Slide numbers are provided so you can refer back.

A rubric is "a statement of purpose or function." As part of the Better Code seminar, we provide simple rubrics to help you write *Better Code*.

Definition

"An *Algorithm* is a process or set of rules to be followed in calculations or other problem-solving operations, especially by a computer." – *New Oxford American Dictionary*

Programming is the construction of algorithms. I often hear, "I don't use or need algorithms." Or "I don't write algorithms." But all coding is the construction of algorithms. Sometimes working on a large project can feel like "plumbing" - just trying to connect components to make them do something. But that *is* creating an algorithm.

Often developers do not understand the algorithm they create.

[clarify how plumbing is creating an algorithm.]

A Simple Algorithm

```
int r = a < b ? a : b;
```


Consider this line of code <click>
This is not a trick question. <wait for answers>
Are you sure? <pause> When I asked, did you have to think about it and double-check?

A Simple Algorithm

```
int r = a < b ? a : b;
```

- What does this line of code do?

A Simple Algorithm

```
// r is the minimum of `a` and `b`
int r = a < b ? a : b;
```


Does a comment help you understand it? Maybe a little?

A Simple Algorithm

```
int r = min(a, b);
```


Is this more clear?

Functions are often ignored but are our most helpful abstraction for constructing software. We frequently focus on type hierarchies and object networks and ignore the basic function building block. In this talk, we're going to explore functions.

Factoring out simple algorithms can significantly impact readability, even for simple lines of code. A comment is not required where the function is used.

Minimum

```
/// returns the minimum of `a` and `b`
int min(int a, int b) {
    return a < b ? a : b;
}
```

Adobe

You can write this comment once - or you can write the comment every time you compute the minimum.

Functions name algorithms. The last seminar introduced contracts to specify functions. Postconditions define the semantics or what the function does. Preconditions, not just the parameter types, define the domain of the operation. Many functions are *partial,* and the domain of a partial function is the values over which the function is defined.

Our `min()` function has no preconditions, which is another way of saying the domain of `min()` is the set of values representable by a pair of `int` types.

We state the postcondition in our specification - associating meaning with the name.

We are defining a vocabulary. We should avoid “making up words” and instead use established names within our domain if the semantics of our operation match.

`min()` is a well-established name for the minimum function. This justifies the use of the abbreviation.

Even for a one-line, trivial operation, the name and associated semantics can make the usage easier to reason about.

Minimum

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```

When implementing an algorithm, we need to reason through each statement

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- The preconditions of each statement must be satisfied by the statements before

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When implementing an algorithm, we need to reason through each statement

- The preconditions of each statement must be satisfied by the statements before
  - Or implied by the preconditions of the algorithm
- The postconditions for the algorithm must follow from the sequence of statements

Minimum

```
/// returns the minimum of `a` and `b`
int min(int a, int b);
```

Functions allow us to build a vocabulary focused on semantics.

After we have defined our function and are sure it is correct, we no longer have to worry about the implementation.

There is a myth that a limited vocabulary makes code easier to read - but this comes at the expense of limiting the ability to express ideas simply. A NAND gate is very simple and can describe all computations. But we don't program using only NANDs

Naming Functions

- Operations with the same semantics should have the same name

Follow existing... The C++ standard library has a relatively rich vocabulary. The vocabulary and conventions in languages differ - defer to your language. C++ shouldn't read like Object Pascal. However, if a language lacks a convention, borrow from another before inventing a new term.

Properties... Dictionary definition "an attribute, quality, or characteristic of something." - a non-mutating operation with a single argument.

consider a verb - Example std::list::size(), and adobe::forest::parent().

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  - copular constructions: `is_blue`
  - consider a verb if the complexity is greater than expected

Naming Functions

- For mutating operations, use a verb:

<end>
Omit needless words.

Naming *is* hard. Focus on capturing the semantics and how it reads at the call site. When choosing a name, writing down your declaration and looking at it is not enough. Write usages of the name. Speak the language.

Naming Functions

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  - `intersection(a, b)` not `calculate_intersection(a, b)`

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- Clarity is of the highest priority. Don't construct unnatural verb phrases
  - `intersection(a, b)` not `calculate_intersection(a, b)`
  - `name()` not `get_name()`

Argument Types

- *let:* by const-lvalue-reference

Three basic ideas in argument passing - this is how they reflect in C++; other languages will have a different mapping.

"Small" is "fits in a register." "Expected" means when used in a template.
Many languages don't have a notion of "sink" - develop or borrow a convention for this use.
Unfortunately, forwarding references have the same syntax as rvalue-references, and disambiguating with *enable_if* or *requires* clauses adds too much complexity.
Prefer return values to out arguments; otherwise, treat as *inout*.
Const in C++ is not transitive - treat it as if it were.

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- *in-out:* by lvalue-reference
  - Prefer sink argument and result to in-out arguments
- spans, views, iterator pairs, and so on are a way to pass a range of objects as if they were a simple argument. The value_type of the range determines if it is a *let* (const) argument or *in-out* (not const), and input ranges are used for *sink* arguments

Argument Types

```
void display(const vector<unique_ptr<widget>>& a) {
    //...
    a[0]->set_name("displayed"); // DONT
    //...
}
```


Don't do this - we'll discuss value semantics more in future seminars, but there is no way to impose transitive const when using reference semantics.

Implicit Preconditions

- Object Lifetimes

Object lifetime can be broken with shared mutable references from shared structures, threads, callbacks, or reentrancy.

The implicit preconditions apply to the arguments passes and to all objects reachable through those arguments. If using reference instead of value semantics, this means the requirements are _deep_.

Implicit Preconditions

- Object Lifetimes
  - The caller must ensure that referenced arguments are valid for the duration of the call

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- Object Lifetimes
  - The caller must ensure that referenced arguments are valid for the duration of the call
  - The callee must copy (or move for sink arguments) an argument to retain it after returning
- Meaning value
  - A meaningless object should not be passed as an argument (i.e., an invalid pointer).

Implicit Preconditions

- Law of Exclusivity

The _Law of Exclusivity_ is borrowed from Swift, and the term was coined by John McCall. C++ does not enforce this rule; it must be manually enforced.

No aliased object under mutation.

The C++ standard library is inconsistent in how it deals with aliasing. Unless aliasing is explicitly allowed, avoid it. Where it is allowed, document (with a comment) any code relying on the behavior.

Nearly every crash is caused by a violation of these implicit preconditions. dereferencing an invalid pointer, using an object after its lifetime, or aliasing a mutable object. Take care! This is a strong argument for why Rust or Val.

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- Law of Exclusivity
  - To modify a variable, exclusive access to that variable is required

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```
vector a{0, 0, 1, 0, 1 };
erase(a, a[0]);
display(a);

{ 1, 0 }
```

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  - To modify a variable, exclusive access to that variable is required
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```
vector a{0, 0, 1, 0, 1 };
erase(a, copy(a[0]));
display(a);

{ 1, 1 }
```

Implicit Postconditions

- Any internal references to (possibly) remote parts held by the caller to an object are assumed to be invalided on return from a mutating operation

Internal references include pointers, iterators, even indices, etc.
Unless the container docs specifically say the iterator is not invalidated, assume it is. Reliance on a class guarantee for reference stability should be noted in a comment at the use site.

The reference returned from vector::back is good until the vector is modified or its lifetime ends

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```
container<int> a{ 0, 1, 2, 3 };
auto f = begin(a);
a.push_back(5);
// `f` is now invalid and cannot be used
```

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- A returned reference must be to one (or a part of one) of the arguments to the function and is valid until the argument is modified or its lifetime ends
  - Example: the reference returned from `vector::back()`

Trivial vs Non-Trivial Algorithms

- A *trivial* algorithm does not require iteration

iteration and recursion and interchangeable - from now on we will just call it "iteration" but statements apply to both.

Trivial vs Non-Trivial Algorithms

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  - Examples: swap(), exchange(), min(), max(), clamp(), tolower()…

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Trivial vs Non-Trivial Algorithms

- A *trivial* algorithm does not require iteration
  - Examples: swap(), exchange(), min(), max(), clamp(), tolower()…
- A *non-trivial* algorithm requires iteration
  - iteration may be implemented as a loop or recursion

Reasoning About Iteration

A finite decreasing property - there must be a mapping of the loop onto natural numbers. You may not know the numbers - but you must prove the mapping exists and that the numbers are decreasing.

Reasoning About Iteration

- To show that a loop or recursion is correct, we need to demonstrate two things:

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- To show that a loop or recursion is correct, we need to demonstrate two things:
  - An invariant that holds at the start of the iteration and after each step
  - A finite decreasing property where termination happens when the property is zero
- The postcondition of the iteration is the above invariant when the decreasing property reaches zero

Remove

We used `erase` a moment ago. erase is built using the `remove()` algorithm. If you have tried to roll your code to erase elements from a container, you might know it can be tricky. Erasing each element going forward gets complex because positions keep moving. Going backward and erasing each element is more straightforward, but both approaches are quadratic. Let's build the remove algorithm to see how to do it.
In order

Remove

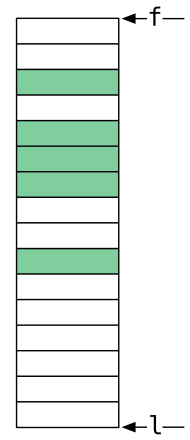
```
/**
    Removes values equal to `a` in the range `[f, l)`.

    \return the position, `b`, such that `[f, b)` contains all the
        values in `[f, l)` not equal to `a`

    values in `[b, l)` are unspecified
*/

template <std::forward_iterator I, class T>
auto remove(I f, I l, const T& a) -> I;
```

Remove

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template <std::forward_iterator I, class T>
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```


Say "in order" when reading the invariant

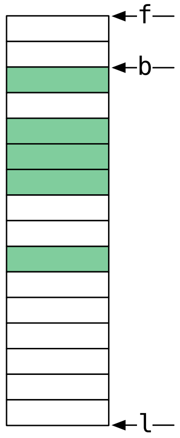
[At end, reread the invariant and decreasing]

Because at termination p equals l, it follow that `[f, b)` contains all the values in `[f, l)` not equal to `a`.

Remove


```
template <std::forward_iterator I, class T>
auto remove(I f, I l, const T& a) -> I {
    auto b{find(f, l, a)};
```

Remove




```
template <std::forward_iterator I, class T>
auto remove(I f, I l, const T& a) -> I {
    auto b{find(f, l, a)};
    if (b == l) return b;
```

Remove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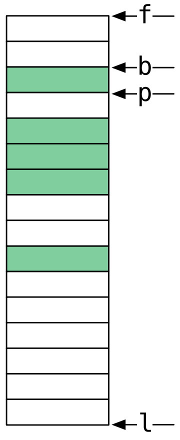
template <std::forward_iterator I, class T>
auto remove(I f, I l, const T& a) -> I {
    auto b{find(f, l, a)};
    if (b == l) return b;
    auto p{next(b)};
```

Remove

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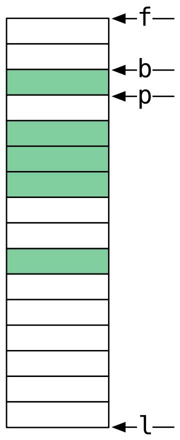
auto remove(I f, I l, const T& a) -> I {
    auto b{find(f, l, a)};
    if (b == l) return b;
    auto p{next(b)};
    // invariant: `[f, b)` contain all the
```

Remove

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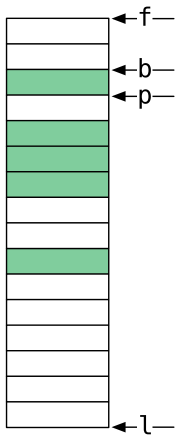


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    auto p{next(b)};
    // invariant: `[f, b)` contain all the
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    // decreasing: `distance(p, l)`
```

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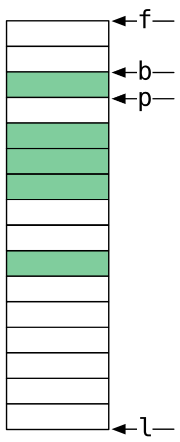
    while (p != l) {
```

Remove

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    while (p != l) {
        if (*p != a) {
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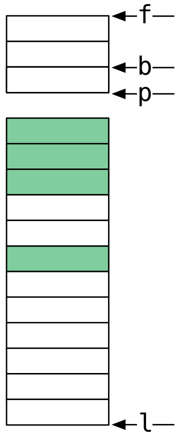


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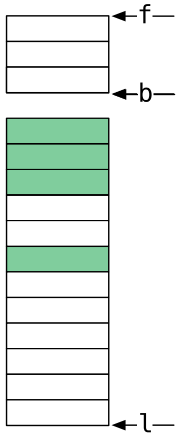
    while (p != l) {
        if (*p != a) {
            *b = std::move(*p);
```

Remove

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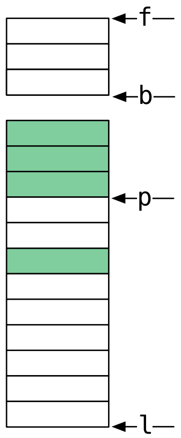


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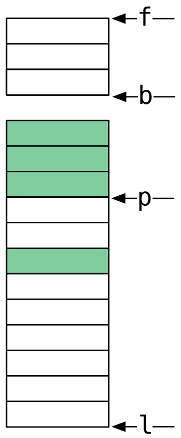


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    while (p != l) {
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            *b = std::move(*p);
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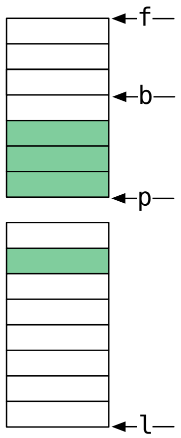
```

Remove

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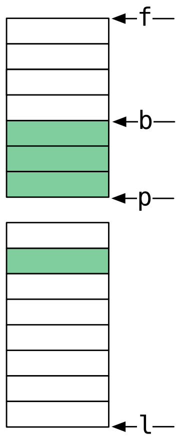


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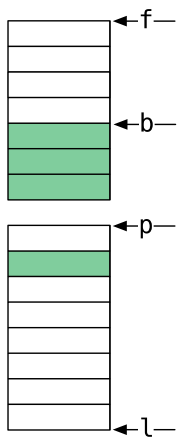


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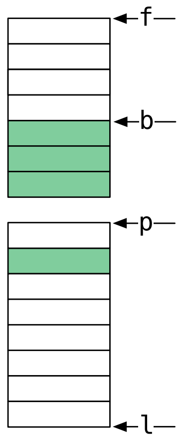


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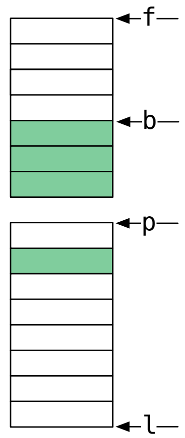


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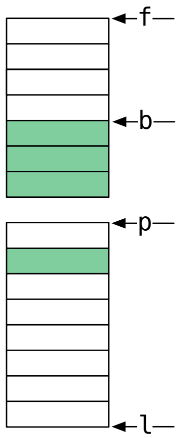


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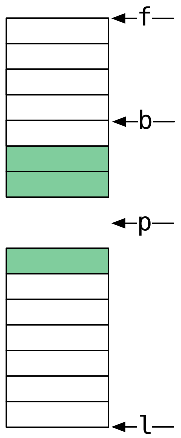


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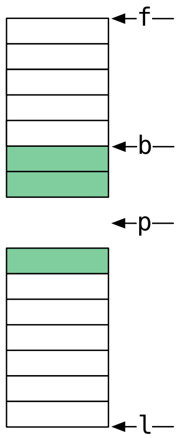


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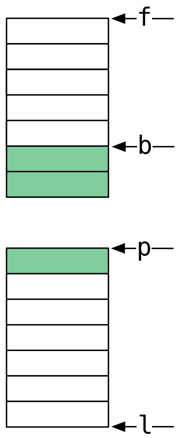


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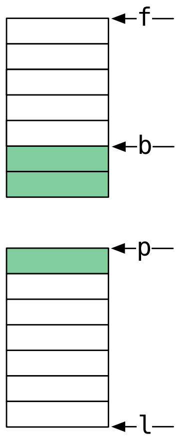


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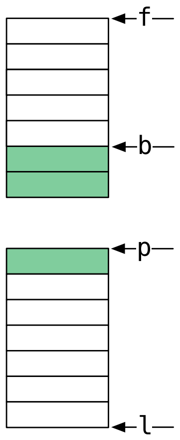


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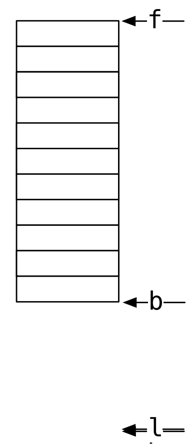


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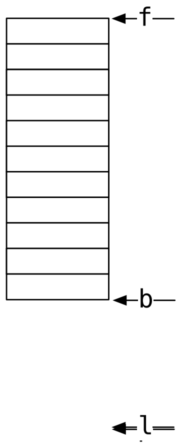
Remove

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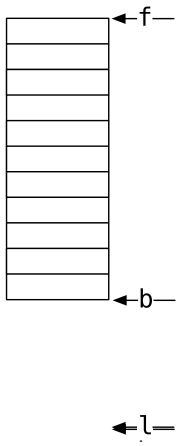


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            *b = std::move(*p);
            ++b;
        }
        ++p;
    }
    return b;
}
```

Remove

```
/**
    Removes values equal to `a` in the range `[f, l)`.

    \return the position, `b`, such that `[f, b)` contains all the
        values in `[f, l)` not equal to `a`

    values in `[b, l)` are unspecified
*/

template <std::forward_iterator I, class T>
auto remove(I f, I l, const T& a) -> I;
```


in order

Sequences

- For a sequence of *n* elements, there are *n + 1* positions

Iteration and recursion imply some form of sequencing. It is essential to understand the properties of sequences for reasoning about loops and iterations.
A closed interval cannot represent an empty interval and is missing one position.
An open interval has one extra position. In an open interval, `f` and `l ` cannot be equal. The empty range of discrete elements is (f, f + 1). Open and closed intervals are mathematic constructs and are most helpful when dealing with continuous values.

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Sequences

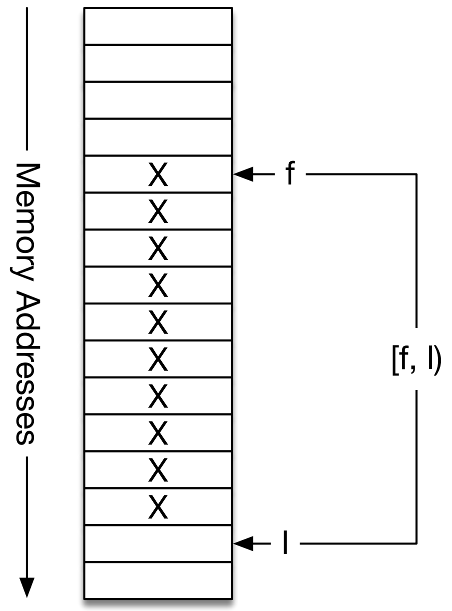
- For a sequence of *n* elements, there are *n + 1* positions
- Ways to represent a range of elements
  - Closed interval [f, l]
  - Open interval (f, l)
  - Half-open interval [f, l)
    - By strong convention, open on the right

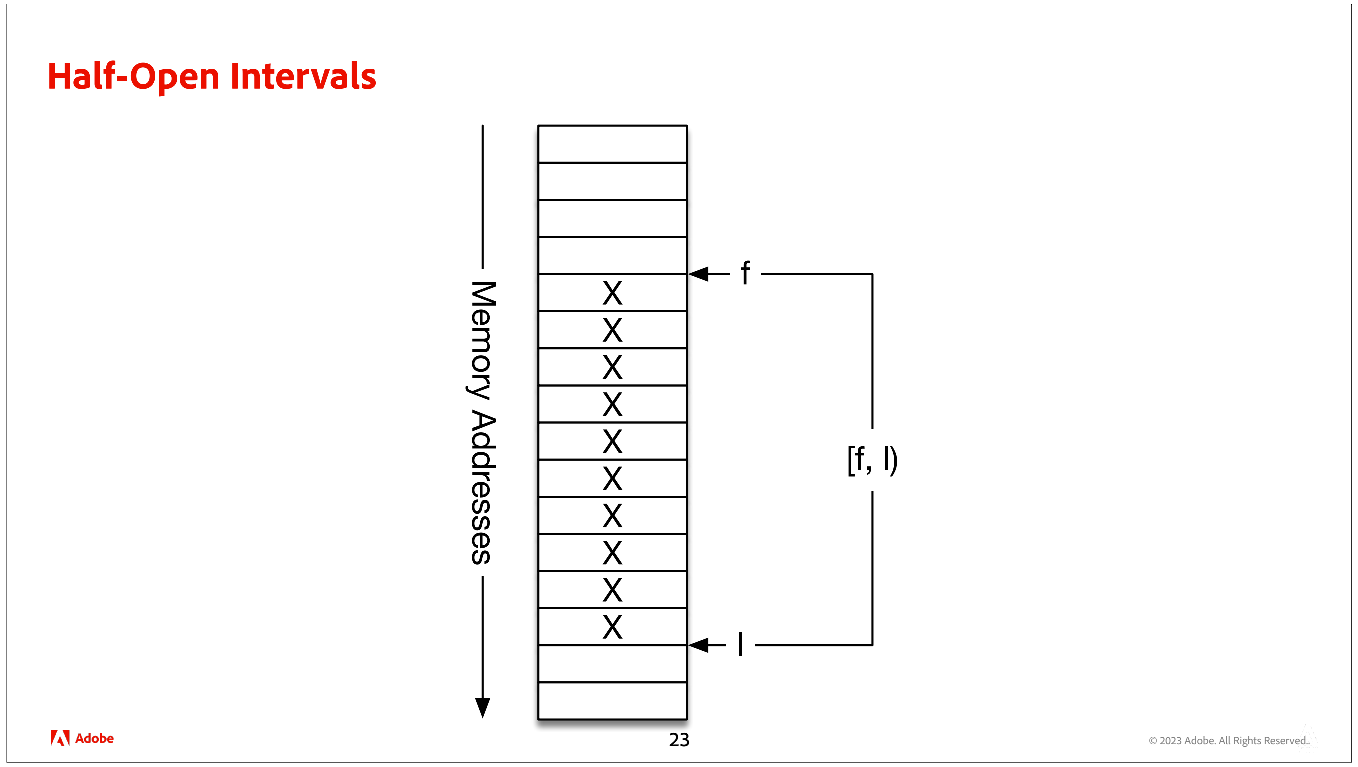
Half-Open Intervals

- *[p, p)* represents an empty range at position p
  - All empty ranges are not equal
- Cannot express the last item in a set with positions of the same set type
  - i.e., `[INT_MIN, INT_MAX]` is not expressible as a half-open interval with type `int`
- Think of the positions as the lines between the elements

Or fence posts.

Half-Open Intervals





first and *last* or *begin()* and *end()* are the first and last *positions*, not the first and last elements.

Half-Open Intervals

- In this model, there is a symmetry with reverse ranges *(l, f]*
  - The dereference operation is asymmetric. dereferencing at a position *p* is the value in *[p, p + 1)*
- Half-open intervals avoid off-by-one errors and confusion about *before* or *after*
- In C and C++, half-open intervals are built into the language. For any object, `a`, `&a` is a pointer to the object, and `&a + 1` is a valid pointer but may not be dereferenceable.
  - Any object can be treated as a range of one element

```
int a{42};
copy(&a, &a + 1, ostream_iterator<int>(cout));
42
```


Alex Stepanov (the creator of STL) would like "while first does not equal last" engraved on his tombstone.

Half-Open Intervals

- Half-open intervals can be represented in a variety of forms

Positions could be pointers, iterators, indices...

Half-Open Intervals

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  - pair of positions: *[f, l)*

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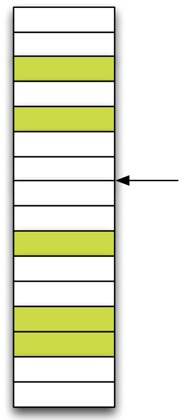
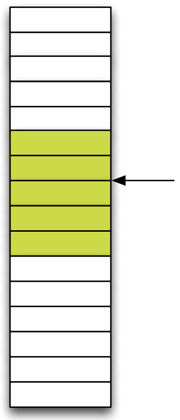
  - position and sentinel: *[f, is_sentinel)*, i.e. NTBS
  - unbounded: *[f, ...)*, limit is dependent on an extrinsic relationship
    - i.e., the range is require to be the same length or greater than another range

Gather

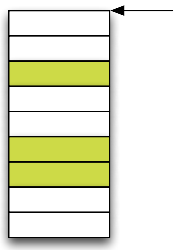


[Need a lead in about why we are composing algorithms. Fix the contracts in this section. Add recursion invariants to stablepartition.]

Gather



Composing Algorithms - Gather



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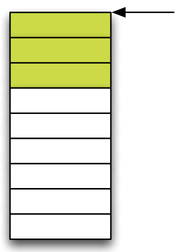

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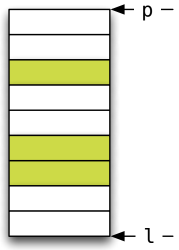
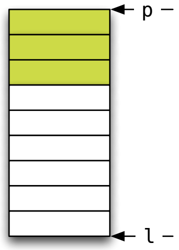



```
stable_partition(p, l, s)
```

Composing Algorithms - Gather

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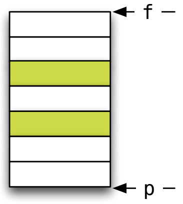
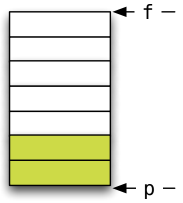
Composing Algorithms - Gather




```
stable_partition(f, p, not1(s))
```

Composing Algorithms - Gather

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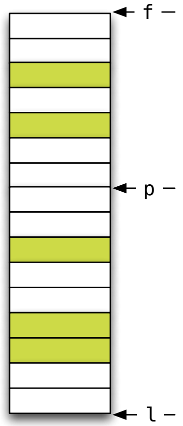
Composing Algorithms - Gather

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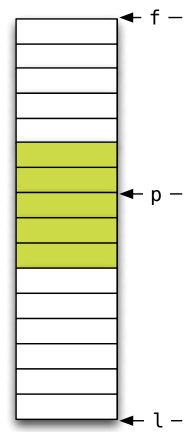
stable_partition(f, p, not1(s))
stable_partition(p, l, s)
```

Composing Algorithms - Gather

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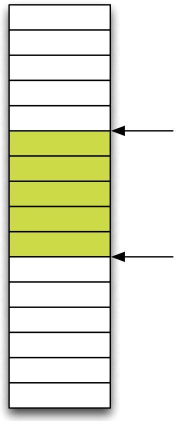

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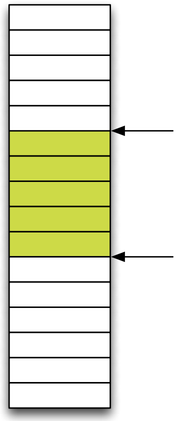
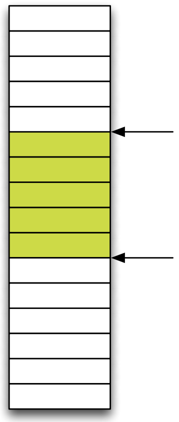


```
return { stable_partition(f, p, not1(s)),
         stable_partition(p, l, s) };
```

Composing Algorithms - Gather



```
/// Gather elements in [f, l) satisfying s at p
/// and returns range containing those elements
/// p is within the result

template <class I, // BidirectionalIterator
          class S> // UnaryPredicate
auto gather(I f, I l, I p, S s) -> pair<I, I>
{
    return { stable_partition(f, p, not1(s)),
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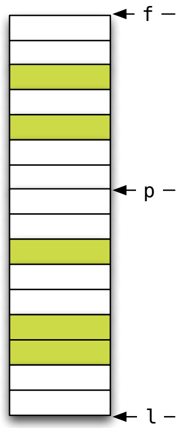
}
```

Composing Algorithms - Gather

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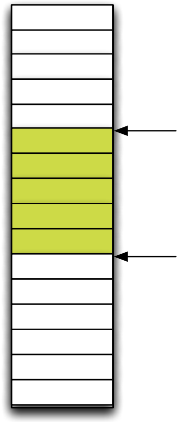

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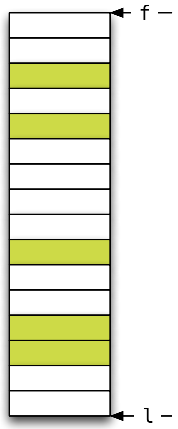
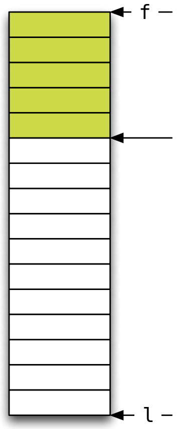
```

Composing Algorithms - Stable Partition



I presented the gather algorithm at a user group meeting. Jon Kalb commented after that, "it was pretty, but few algorithms compose like that." But this isn't true – most algorithms are simple compositions of other algorithms. Let's look at how to implement stable partition

Composing Algorithms - Stable Partition



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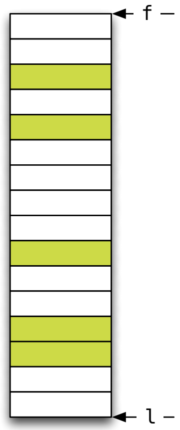

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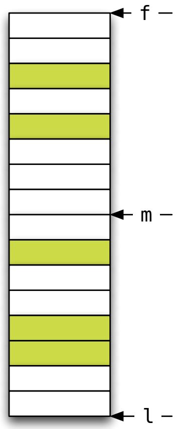

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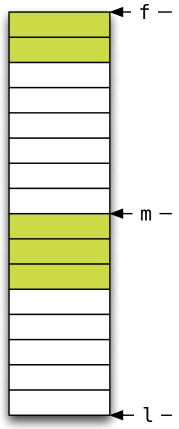

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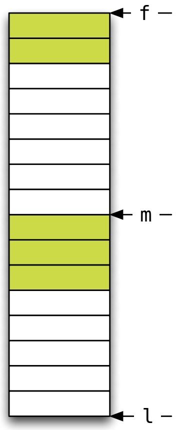


```
stable_partition(f, m, p)

stable_partition(m, l, p)
```

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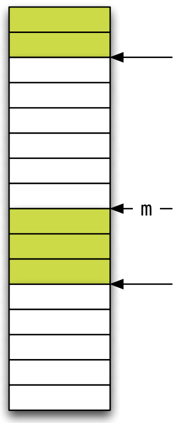


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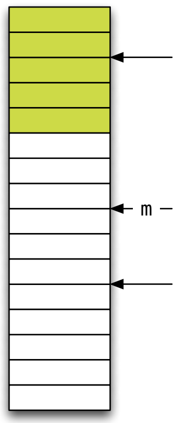


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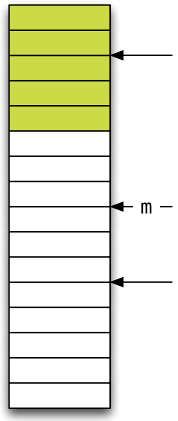
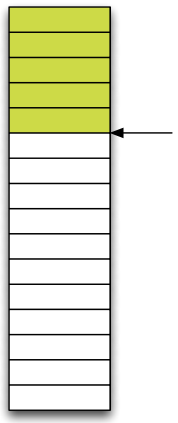



```
rotate(stable_partition(f, m, p),
       m,
       stable_partition(m, l, p));
```

Composing Algorithms - Stable Partition



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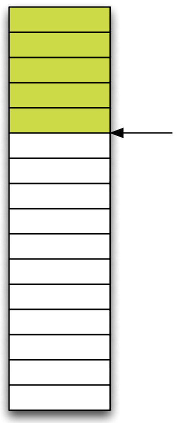

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return rotate(stable_partition(f, m, p),
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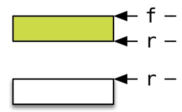


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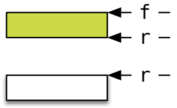


```
if (n == 1) return f + p(*f);

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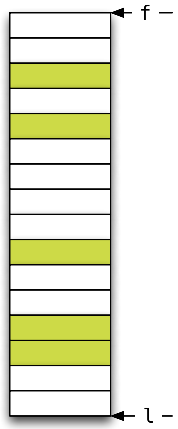
Composing Algorithms - Stable Partition

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I did not include a contract here because stable_partition is a standard algorithm, and there wasn't space on the slide.

Interestingly, the predicate is only evaluated once on each element before the element is moved. Then everything is rotated into position.
A stable partition is implemented with *rotate()*. Rotate is a fascinating algorithm that could fill an hour, but one implementation is three reverses.
Reverse is iterative calls to swap. Many STL algorithms, including stable partition, exist to implement in-place stable sort.

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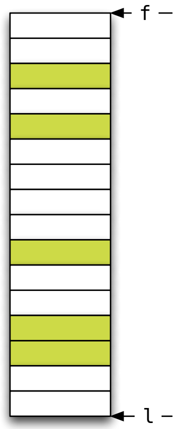


```
template <class I, // ForwardIterator
          class P> // UnaryPredicate
auto stable_partition(I f, I l, P p) -> I
{
    auto n = l - f;
    if (n == 0) return f;
    if (n == 1) return f + p(*f);

    auto m = f + (n / 2);

    return rotate(stable_partition(f, m, p),
                  m,
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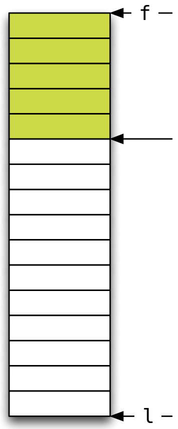


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Much More

- complexity and efficiency

Sort maps the relationship
We'll talk more about structured data and relationships in future seminars

Much More

- complexity and efficiency
- sorting and heap algorithms

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- sorting and heap algorithms
  - encoding relationships between properties into structural relationships to create *structured data*

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  - encoding relationships between properties into structural relationships to create *structured data*
  - i.e., *a* < *b implies position(a)* < *position(b)*

Why No Raw Loops?

- Difficult to reason about and difficult to prove post conditions

This brings us back to our rubric

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- Error prone and likely to fail under non-obvious conditions

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- Introduce non-obvious performance problems
- Complicates reasoning about the surrounding code

Alternatives to Raw Loops

- Use an existing algorithm

Most of the standard algorithms have all been machine proven to be correct - this is not Adobe's policy publishing provides the same bonus as a patent bonus and some legal protections.

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∅ Patents

Q & A

About the artist

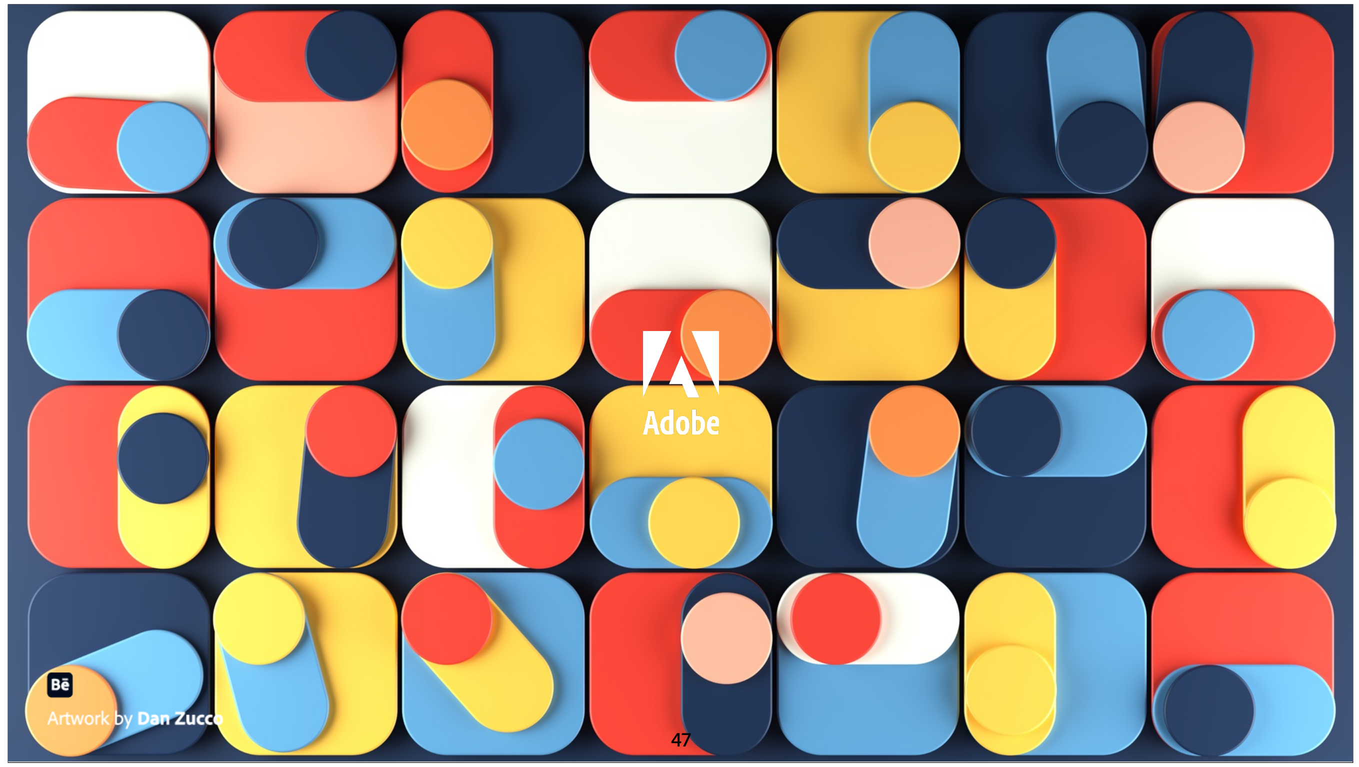
Dan Zucco

London-based 3D art and motion director Dan Zucco creates repeating 2D patterns and brings them to life as 3D animated loops. Inspired by architecture, music, modern art, and generative design, he often starts in Adobe Illustrator and builds his animations using Adobe After Effects and Cinema 4D. Zucco's objective for this piece was to create a geometric design that felt like it could have an infinite number of arrangements.

Made with

Adobe Illustrator

Adobe After Effects


Adobe
Artwork by Dan Zucco